Math 494
Midterm Examination 2 – due April 6, 2011

Name ________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may use calculators or computer algebra systems; and refer to your notes, to the textbooks Hogg-McKean-Craig and Grinstead-Snell (but not other books), or to Wikipedia (but not other websites, including Google). You may not discuss the exam problems with anyone other than Russ.

1. (1 points each) True/False. Please read the statements carefully, as no partial credit will be given.

   (a) ____ If Y is an mle for θ, then e^Y is an mle for e^θ.

   (b) ____ If Y is an unbiased estimator for θ, then e^Y is an unbiased estimator for e^θ.

   (c) ____ If Y is an efficient estimator for θ, then e^Y is an efficient estimator for e^θ.

   (d) ____ If Y is the minimum value from a sample of size n from some distribution, then $E(Y) = \xi_{\frac{1}{n+1}}$.

   (e) ____ If M is the sample median for an iid sample (of odd size) from an unknown distribution, then $P(M < \xi_{0.5}) = \frac{1}{2}$.

   (f) ____ If $f(x;\theta)$ is the pdf of a uniform random variable on an interval of length 2, with an unknown mean (parametrized by θ), then f satisfies (R1).

   (g) ____ Given an iid sample from an $N(\mu, 1)$ distribution (where μ is unknown), there is a unique unbiased estimator for μ.

   (h) ____ If $f(x;\theta)$ is the pdf for $N(\theta^3, 1)$, then f satisfies (R0).

2. You are still examining the unknown radioactive material from Exam 1. The time X between radioactive emissions is an exponential random variable, with unknown expected parameter μ.

   (a) (4 points) Find the Fisher information of X with respect to parameter μ. (You may assume condition (R4) is satisfied).

   (b) (4 points) We have shown that $\overline{X}$ is an unbiased estimator for μ. Is $\overline{X}$ efficient? If not, find its efficiency.

   (c) (3 points) You take 3 iid samples x_1, x_2, x_3 from X and calculate the order statistics $y_1 < y_2 < y_3$. Find a bounded confidence interval for ξ_p, where $p = \frac{1}{2}$ or $\frac{1}{e}$.
   (Obviously, you won't be able to get 95% confidence with $n = 3$! Work at the highest confidence level that you are able.)

   (d) (1 point) Compare your confidence intervals from part (c) with the one from the Exam 1, Problem 2b solutions. Explain in 2-4 sentences which one you prefer.

3. (5 points) Find all mles for the following family of pdfs:

$$f(x;\theta) = \begin{cases} \frac{2}{\theta} x e^{-x^2/\theta} & \text{for } x, \theta \in (0, \infty) \\ 0 & \text{else.} \end{cases}$$

4. (5 points) Let X be a discrete random variable with a countably infinite range, and let $\varphi(t)$ be a continuous convex function. Assume that $E(X)$ and $E(\varphi(X))$ both exist. Prove Jensen's inequality for X and φ.
   Hint: It may be easier to think about if you order the range of X such that $p(a_1) \geq p(a_2) \geq p(a_3) \geq \dots$.