Math 132
Midterm Examination 2 – March 5, 2012

6 multiple choice, 4 long answer. 100 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Part I of the exam is multiple choice, while Part II is long answer.

Part I Instructions: If you do not have a pencil to fill out your answer card, please ask to borrow one from your proctor. Write your Student ID number on the six blank lines on the top of your answer card, and shade in the corresponding bubbles to the right of each digit.
Fill in the bubble corresponding to each of the following 6 questions. Each is worth 4 points. On Part I, no partial credit will be given.

1. Find the Trapezoid Rule approximation using 4 subintervals of

$$\int_{-1}^{1} x^2\, dx.$$

(a) 0
(b) 1/4
(c) 1/3
(d) 1/2
(e) 2/3
(f) 3/4
(g) 1
(h) 5/4
(i) 4/3
(j) 3/2

2. Find the Simpson's Rule approximation using 4 subintervals of

$$\int_{-1}^{1} x^2\, dx.$$

   (a) 0
   (b) 1/4
   (c) 1/3
   (d) 1/2
   (e) 2/3
   (f) 3/4
   (g) 1
   (h) 5/4
   (i) 4/3
   (j) 3/2

3. Consider the system consisting of 3 point masses:
   10 kg at $(3, -1)$
   20 kg at $(2, 10)$
   100 kg at $(1, 0)$
   The center of mass is:

   (a) $(0, 0)$
   (b) $(\frac{2}{15}, \frac{3}{15})$
   (c) $(\frac{2}{13}, \frac{3}{13})$
   (d) $(\frac{6}{15}, \frac{9}{15})$
   (e) $(\frac{6}{13}, \frac{9}{13})$
   (f) $(\frac{17}{15}, \frac{19}{15})$
   (g) $(\frac{17}{13}, \frac{19}{13})$
   (h) $(2, 3)$
   (i) None of the above/does not exist.

4. Simpson's Rule applied to the integral $\int_1^e \frac{1}{x}\,dx$ with $n = 20$ will be closest to:

(a) 0
(b) 1/10
(c) 2/10
(d) 3/10
(e) 4/10
(f) 5/10
(g) 6/10
(h) 7/10
(i) 8/10
(j) 9/10
(k) 1

5. Find the average value of $\sin x$ over the interval $[0, \pi]$.

(a) 0
(b) 1/2
(c) $\pi/5$
(d) $2/\pi$
(e) 2/3
(f) $\pi/4$
(g) $3/\pi$
(h) 1
(i) None of the above/does not exist.

6. The decay of a certain radioactive isotope of the element rabbitonium is governed by the differential equation $y' = -ky$. At $t = 0$ you have 300 mg of radioactive rabbitonium. At $t = 45$ minutes, you are left with only 100 mg of radioactive rabbitonium. Then k is ______ per minute.

   (a) $\ln 2/15$.
   (b) $\ln 3/15$.
   (c) $\ln 2/30$.
   (d) $\ln 3/30$.
   (e) $\ln 2/45$.
   (f) $\ln 3/45$.
   (g) $\ln 2/60$.
   (h) $\ln 3/60$.
   (i) None of the above.

Part II Instructions: Answer the following on the exam sheet, showing all your work. Correct answers without correct supporting work may not receive full credit. You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.
Please put your name and student id number on **each page** of Part II now.

1. Differential equations

   (a) (8 points) Solve the differential equation $y' = x + xy$ subject to the initial condition $y(0) = 5$.

   (b) (8 points) At time $t = 0$, there is 1000 liters of water in a tank, with 80 kg of salt dissolved in it. Distilled water flows into the tank at 10 L/min, and water flows out of the tank at the same rate. The tank is continually stirred, and the salt is kept mixed evenly through the tank.
   Set up a differential equation (you needn't solve it) for the mass of salt in the tank at time t. (Your answer should be of the form $y' =$ ____.)

2. Arc lengths and approximate integration

(a) (6 points) Set up a definite integral representing the length of the curve $y = x^3$ between $x = 0$ and $x = 4$.

(b) (10 points) The first several derivatives of $f(x) = \sqrt{1+x^2}$ are as follows:

$$f'(x) = \frac{x}{\sqrt{1+x^2}}, \qquad f''(x) = \frac{1}{(1+x^2)^{3/2}}, \qquad f^{(3)}(x) = \frac{-3x}{(1+x^2)^{5/2}},$$
$$f^{(4)}(x) = \frac{12x^2-3}{(1+x^2)^{7/2}}, \qquad f^{(5)}(x) = \frac{45x-60x^3}{(x^2+1)^{9/2}}.$$

Find (with justification) an n such that the Simpson's Rule approximation S_n for $\int_{-1}^{4} \sqrt{1+x^2}\,dx$ has error at most 0.001.

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3. Calculations

(a) (6 points) Find an upper bound for $\left|2e^{-(x+1)^2} + 12\sin(x+1)^2\right|$ on the interval $[-3, 3]$.

(b) (7 points) Evaluate $\displaystyle\int x^2 \cos x \, dx$.

(c) (6 points) Evaluate $\displaystyle\int_0^1 \frac{x}{1+x^2} \, dx$.

(d) (6 points) Evaluate $\displaystyle\int_{-1}^1 x \tan^{-1} x \, dx$.

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4. Volumes and centroids

   In both problems on this page, we consider the region between the x-axis and the graph of $y = e^x$ for $0 \leq x \leq 2$.

   (a) (11 points) Find the volume of the solid formed by rotating the given region around the y-axis.

   (b) (8 points) Find the center of mass $\overline{x}$ with respect to x of the solid formed by rotating the given region around the x-axis.
   Half credit will be received for instead finding the center of mass $\overline{x}$ of the given (unrotated) region.