Math 233 Fall 2004

HOMEWORK # 10

(Due : Thursday 12/9 . Either in class or Math Office before 5 PM)

1) Use theorems we've learned to show that for any two points A , B in R^3 , $\int_C z^2\, dx + 2y\, dy + 2xz\, dz$ is the same for any curve C connecting A to B .

2) Find a potential function f(x,y,z) for the vector field $\mathbf{F}(x,y,z) = (e^x\, ln(y))\, \boldsymbol{i} + (\frac{e^x}{y} + sin(z))\, \boldsymbol{j} + (y\cos(z))\, \boldsymbol{k}$.
(The answer without showing how you got it is not sufficient)

3) Let $\mathbf{F} = \nabla(x^3y^2)$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the path connecting (− 1 , 1) to (1 , 1) that consists of C_1 , the line segment from (− 1 , 1) to (0 , 0) , followed by C_2 , the line segment from (0 , 0) to (1 , 1)
a) by finding parametrizations for C_1 and C_2 and evaluating integral , and
b) by using the potential vector , $f(x, y) = x^3y^2$.

4) For what values of the numbers b and c , will **F** be a <u>gradient</u> vector field , where $\mathbf{F} = (y^2 + 2czx)\, \boldsymbol{i} + y(bx + cz)\, \boldsymbol{j} + (y^2 + cx^2)\, \boldsymbol{k}$.

5) Verify the statement of <u>Green's Theorem</u> for $\mathbf{F} = (-x^2y)\, \boldsymbol{i} + (xy^2)\, \boldsymbol{k}$, where C is the circle $x^2 + y^2 = a^2$ travelled in the counterclockwise direction from (a , 0) back to (a , 0) .
(Note : C can be $\mathbf{r} = (a\cos(t))\, \mathbf{i} + (a\sin(t))\, \mathbf{j}$, $0 \le t \le 2\pi$)

6) Apply Green's Theorem to evaluate $\int_C 3y\, dx + 2x\, dy$, where C is a boundary of the region $0 \le x \le \pi$, $0 \le y \le \sin(x)$.