

HOMEWORK #9

(Due : Thursday 12/12 , before 5 PM . Bring it to class or to the Math Office )

- 1) Find the **gradient vector field** for the function  $f(x, y, z) = e^z - \ln(x^2 + y^2)$ .
- 2) Find a formula for the vector field  $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ , given the fact that for all points  $(x, y)$ ,  $\mathbf{F}$  points towards the origin, and  $|\mathbf{F}| = \frac{10}{x^2 + y^2}$ .  
(The region is all points except  $(0, 0)$ .)
- 3) Find the **work done** by the **force**  $\mathbf{F} = 2y\mathbf{i} + 3x\mathbf{j} + (x + y)\mathbf{k}$  in moving object along the curve  $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \frac{1}{6}\mathbf{k}$  where  $0 \leq t \leq 2\pi$ .
- 4) Evaluate  $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$ , where  $\mathbf{F} = x^2\mathbf{i} - y\mathbf{j}$  and  $C$  is the curve  $y = x^2$ , from  $(4, 2)$  to  $(1, -1)$ .
- 5) Find the **flow** of the velocity vector field  $\mathbf{F} = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ , along the curve  $C = C_1 \cup C_2$ , where  $C_1$  is the line segment from  $(1, 0)$  to  $(0, -1)$  and  $C_2$  is the line segment from  $(0, -1)$  to  $(-1, 0)$ .
- 6) Sketch the vector field  $\mathbf{F} = -\frac{y}{\sqrt{x^2 + y^2}}\mathbf{i} + \frac{x}{\sqrt{x^2 + y^2}}\mathbf{j}$ , at the 8 points on the circle  $x^2 + y^2 = 4$ , each point  $\frac{\pi}{4}$  radians apart, starting with  $(2, 0)$ . In the sketch indicate at each vector the  $\mathbf{i}^{th}$  and  $\mathbf{j}^{th}$  components of the vector.