1. (Problem 30, page 40) If \( E \in \mathcal{L} \) and \( 0 < m(E) < \infty \), then for any \( \alpha < 1 \) there exists a nonempty open interval \( I = (a, b) \) such that \( m(E \cap I) > \alpha m(I) \).

(Note: \( \mathcal{L} = \mathcal{M}(\lambda^{*}) \) is the Lebesgue-measurable subsets of \( R^{1} \) and \( m = \lambda^{*} \); see page 37.)

2. (Like problem 26, page 40) Let \( E \in \mathcal{M}(\mu^{*}) \) for a Borel measure \( \mu \) on \( R^{1} \) with \( \mu(E) < \infty \) (writing \( \mu = \mu^{*} \) on \( \mathcal{M}(\mu^{*}) \)). Prove that, for any \( \epsilon > 0 \), there exists a finite union of cells \( A = \bigcup_{i=1}^{n} (a_{i}, b_{i}] \) such that \( \mu(E \triangle A) < \epsilon \).

3. Let \( a_{i}, b_{i} \ (1 \leq i \leq n) \) be real numbers with \( a_{i} \leq b_{i} \ (1 \leq i \leq n) \). Assume that \( \sum_{i=1}^{n} a_{i} = \sum_{i=1}^{n} b_{i} \). Prove that \( a_{i} = b_{i} \) for \( 1 \leq i \leq n \).

4. Let \( F(x) = \lfloor x \rfloor \) be the greatest-integer function on \( R \). Note that \( F(x) \) is increasing and right continuous.

   (i) Are there any points \( a \in R \) for which \( \mu_{F}(\{a\}) > 0 \)? If so, which points \( a \)? What are the corresponding values of \( \mu_{F}(\{a\}) \)?

   (ii) Let \( A = \bigcup_{i=1}^{5} (\frac{i}{2} - \frac{1}{5}, \frac{i}{2} + \frac{1}{10}] \). Find \( \mu_{F}(A) \) and justify your answer. (Hint: Be careful!)

5. Let \( Q \) be the rationals. Set \( (a, b]_{Q} = \{ q \in Q : a < q \leq b \} \) and let \( \Gamma_{Q} = \{ (a, b]_{Q} : a, b \in Q \} \). Define \( \nu(A) \) on \( \Gamma_{Q} \) by \( \nu((a, b]_{Q}) = b - a \).

   (i) Show that \( \Gamma_{Q} \) is a semi-ring of subsets of \( Q \) and that \( \nu \) is finitely additive on \( \Gamma_{Q} \).

   Use \( \nu \) on \( \Gamma_{Q} \) to define the outer measure \( \nu^{*}(E) \) for \( E \subseteq Q \).

   (ii) Show that \( \nu^{*}(\{x\}) = 0 \) for all \( x \in Q \)

   (iii) Show that \( \nu^{*}(E) = 0 \) for all subsets \( E \subseteq Q \).

In particular, it is not true that \( \nu^{*}(A) = \nu(A) \) for all \( A \in \Gamma_{Q} \). While it is true that \( \nu^{*}(A) \) is a measure on the \( \sigma \)-algebra of all subsets of \( Q \), this is not a particularly interesting statement since \( \nu^{*}(A) \) is identically zero.

   (iv) At what step or steps do the proofs of Propositions 1.15 or 1.13 break down?