Math 217 April 10, 2024
Krantz

THIRD MIDTERM EXAM

General Instructions: Read the statement of each problem carefully. If you want full credit on a short-answer problem then you must show your work. If you only write the answer then you will *not* receive full credit.

Be sure to ask questions if anything is unclear. This exam has 12 questions and each question is worth 10 points.

1. Verify that 0 is a regular singular point for the differential equation

$$x^2y'' + x(2x-1)y' - y = 0\,.$$

Now determine the values of m that will make the Frobenius solution valid.

2. Calculate the Fourier series of the function $f(x) = 2x$, $-\pi \leq x \leq \pi$.

3. Calculate the cosine series of the function $g(x) = x$ on the interval $[0, \pi]$.

4. Calculate the Fourier series of the function $h(x) = 3x$ on the interval $[-1, 1]$.

5. Calculate the inverse Laplace transform of the function

$$F(p) = \frac{1}{p^4 - p^2}\,.$$

6. Solve the initial value problem

$$y'' - 6y' + 9y = 0 \quad , \quad y(0) = 0 \ , \ y'(0) = 3 \, .$$

using the Laplace transform.

7. Calculate the convolution of the functions $f(t) = t$ and $g(t) = e^{-t}$.

8. Use the unit impulse function to solve the initial value problem

$$y'' + y' - 2y = t \quad , \quad y(0) = 0 \ , \ y'(0) = 0 \, .$$

9. Explain why the function $f(x) = (\sin x) \cdot (\sin x)$ is even on the interval $[-\pi, \pi]$.

10. Define an inner product on the square integrable functions on $[-\pi, \pi]$ by

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x)\, dx\,.$$

Using this inner product, explain why the functions $\sin jx$ and $\sin kx$ are orthogonal when $j \neq k$.

11. If $F(p)$ is the Laplace transform of the function f, then explain why

$$\frac{d}{dp}F(p) = L[-xf(x)]\,.$$

12. Calculate

$$L[x \cos x]\,.$$