1. (10 points) Find the equation of the tangent plane to the surface \( x^2 - y^2 + z^4 = 12 \) at the point \((0, 2, 2)\).

Let \( F(x, y, z) = x^2 - y^2 + z^4 \). Then the normal \( \nabla F = \langle 2x, -2y, 4z^3 \rangle \). At the point \((0, 2, 2)\) this is \( \langle 0, -4, 32 \rangle \). Then the equation of the plane is \( \langle 0, -4, 32 \rangle \cdot \langle x - 0, y - 2, z - 2 \rangle = 0 \)

or \( 0x - 4y + 32z = 56 \).
2. (8 points) Use polar coordinates to calculate the area inside the curve \( r = \sin \theta \).

\[
\int_0^\pi \int_0^{\sin \theta} r \, dr \, d\theta = \int_0^\pi \frac{r^2}{2} \bigg|_0^{\sin \theta} \, d\theta \\
= \int_0^\pi \frac{\sin^2 \theta}{2} \, d\theta = \frac{\pi}{4} - \frac{\sin 2\theta}{8} \bigg|_0^{\pi/2} \\
= \frac{\pi}{4} - \frac{1}{8} \left(0 - 0\right) = \frac{\pi}{4}.
\]
3. (4 points) Change the order of integration in order to evaluate the integral
\[
\int_{0}^{\pi/2} \int_{x}^{\pi/2} \frac{\sin y}{y} \, dy \, dx.
\]
\[
= \int_{0}^{\pi/2} \left[ \frac{\sin y}{y} \right]_{x}^{\pi/2} \, dx
= \int_{0}^{\pi/2} \frac{\sin y}{y} \, dy
= \left[ -\cos y \right]_{x}^{\pi/2}
= -\cos \frac{\pi}{2} + 0 + 1 = 1
\]

4. (6 points) The center of curvature for the curve \( r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + tk \) at the point \((-1, 0, \pi)\) is given by

(a) \((0, 1, \pi)\)
(b) \((\pi, 1, 0)\)
(c) \((0, -1, \pi)\)
(d) \((1, 0, \pi)\)
(e) \((-1, 0, \pi)\)

\[ r'(t) = -\sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k} \]
\[ r''(t) = -\cos t \mathbf{i} - \sin t \mathbf{j} + 0 \mathbf{k} \]
\[ r' \times r'' = \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin t & -\cos t & 1 \\ -\cos t & -\sin t & 0 \end{pmatrix} = \mathbf{i}(\sin t) - \mathbf{j}(\cos t) + \mathbf{k}(\sin^2 t + \cos^2 t) = \mathbf{i}\sin t - \mathbf{j}\cos t + \mathbf{k} \]
\[ ||r' \times r''|| = 3 \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{3} \]
\[ ||r'|| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2} \]
\[ k = \frac{||r' \times r''||}{||r'||^3} = \frac{\sqrt{3}}{\sqrt{2}^3} = \frac{1}{2} \]
So, \( \rho = 2 \).
5. (6 points) **TRUE** or **FALSE**: The vector field \( \mathbf{F}(x, y, z) = yz \mathbf{i} + xz \mathbf{j} + xy \mathbf{k} \) is conservative. Put your answer here:

\[
\mathbf{F} = \nabla u,
\]

where \( u(x, y, z) = xyz \).

So \( \mathbf{F} \) is **conservative**.
6. (8 points) TRUE or FALSE: The vector field \( \mathbf{F}(x, y) = x \cos y \mathbf{i} - y \sin x \mathbf{j} \) is path independent. Put your answer here:

\[
\begin{align*}
\frac{\partial}{\partial y} (x \cos y) &= -x \sin y \\
\frac{\partial}{\partial x} (-y \sin x) &= -y \cos x \\
\end{align*}
\]
These are unequal so \( \mathbf{F} \) is not closed. Therefore \( \mathbf{F} \) is not conservative. So it is not path independent.
7. (6 points) Sketch the graph of the function \( x = y^2 + z^2 \) on this three-dimensional set of axes. Be sure to exhibit your calculations of level sets, and be sure to label the level sets in your graph.

\[
\begin{align*}
x &= 1 \\
y &= y^2 + z^2 \\
z &= y^2 + z^2 \\
x &= y \\
x &= 0
\end{align*}
\]

8. (8 points) The parametric equations for the tangent line to the curve \( \mathbf{r}(t) = t^3 \mathbf{i} - t^2 \mathbf{j} + 4t \mathbf{k} \) at the point \((8, -4, 8)\) are given by (circle your answer)

(a) \[
\begin{align*}
x &= 3 + 3t \\
y &= -8 - 10t \\
z &= 6 + 2t
\end{align*}
\]
(b) 

\[
x = xx \\
y = yy \\
z = zz 
\]

(c) 

\[
x = 8 + 12t \\
y = -4 - 4t \\
z = 8 + 4t 
\]

(d) 

\[
x = 4 + 4t \\
y = -8 + 12t \\
z = 6 + 3t 
\]

\[
r'(t) = 3t^2 \hat{i} - 2t \hat{j} + 4 \hat{k} 
\]

At \( t = 2 \) this is \( 12\hat{i} - 4\hat{j} + 4\hat{k}. \)

So the line is 

\[
x = 8 + 12t \\
y = -4 - 4t \\
z = 8 + 4t 
\]
9. (6 points) The curl of the vector field \( \mathbf{F}(x, y, z) = x \sin y \mathbf{i} + y \cos x \mathbf{j} - \tan z \mathbf{k} \) at the point \((\pi/2, 2\pi, \pi)\) is given by

(a) \( \pi \mathbf{i} - \pi \mathbf{k} \)
(b) \( 4\mathbf{i} - \pi \mathbf{j} + \pi /2 \mathbf{k} \)
(c) \( (-5\pi /2) \mathbf{k} \)
(d) \( \pi \mathbf{j} - 3 \mathbf{k} \)

\[
\text{curl} \ \mathbf{F} = \text{det} \begin{pmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
x \sin y & y \cos x & -\tan z
\end{pmatrix}
\]

\[
= \mathbf{i} (0 - 0) - \mathbf{j} (0 - 0) + \mathbf{k} (-y \sin x - x \cos y),
\]

At \((\pi/2, 2\pi, \pi)\),

\[
\text{curl} \ \mathbf{F} = \mathbf{k} (-2\pi \cdot 1 - \pi /2 \cdot 1) = -5\pi /2.
\]

10. (8 points) The divergence of the vector field \( \mathbf{F}(x, y, z) = xz \mathbf{i} + yz \mathbf{j} + yx \mathbf{k} \) at the point \((1, 2, 3)\) is given by

(a) 4
(b) 6
(c) -3
(d) 5

\[
\text{div} \ \mathbf{F} = \frac{\partial}{\partial x} (xz) + \frac{\partial}{\partial y} (yz) + \frac{\partial}{\partial z} (yx)
\]

\[
= z + z = 2z.
\]

At \((1, 2, 3)\) we get \(\text{div} \ \mathbf{F} = 6\).
11. (6 points) Use Green’s theorem to calculate \( \int_C \mathbf{F} \, dr \) for \( \mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} \) and \( \mathbf{F}(x, y) = -yi + xj \).

\[
\int_C \mathbf{F} \, dr = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dA = \iint_{\text{disc}} (1 + 1) \, dA
\]

\[= 2\pi.
\]

12. (6 points) TRUE or FALSE: The vector field \( \mathbf{F}(x, y) = xy^2 \mathbf{i} - yx^2 \mathbf{j} \) is closed. Put your answer here:

\[
\frac{\partial}{\partial y} (xy^2) = 2xy
\]

\[
\frac{\partial}{\partial x} (y^2x) = -2xy.
\]

These are unequal so the vector field is not closed.
13. (6 points) Sketch the graph of the polar function $r = \cos 3\theta$ on this polar set of axes.
14. (6 points) Find and identify the critical points of the function \( f(x, y) = x^2 + 4y^2 - 6xy + 8 \).

\[
\begin{align*}
\nabla f &= \langle 2x + 6y, 8y + 6x \rangle \\
2x + 6y &= 0 \\
8y + 6x &= 0 \\
x &= 0, y = 0 \\
\det \left( \begin{array}{cc} 2 & 6 \\ 6 & 8 \end{array} \right) &= 16 - 36 < 0 \\

(0, 0) \text{ is a saddle point.}
\end{align*}
\]

15. (6 points) The maximum and minimum values of the function \( f(x, y) = 4x - 3y \) subject to the constraint \( g(x, y) = x^2 + y^2 = 4 \) are:

(a) maximum = 20, minimum = -20
(b) maximum = \( 50/\sqrt{17} \), minimum = -50/\sqrt{17}
(c) maximum = 10, minimum = -10
(d) maximum = 10/\sqrt{17}, minimum = -10/\sqrt{17}

\[
\nabla f = \lambda \nabla g \Rightarrow \langle y, -3 \rangle = \lambda \langle 2x, 2y \rangle \Rightarrow 4 = 2\lambda x, -3 = 2\lambda y.
\]

So \( x = 2/\lambda, y = -3/(2\lambda) \). Substitute into the constraint equation to get

\[
\left( \frac{2}{\lambda} \right)^2 + \left( \frac{-3}{2\lambda} \right)^2 = 1
\]

\[
16 + 9 = 16\lambda^2
\]

\[
\frac{25}{16} = \lambda^2
\]

\[
\lambda = \pm 5/4
\]

\[
\lambda = \frac{5}{4} \Rightarrow x = \frac{8}{5}, y = -\frac{6}{5}
\]

\[
\lambda = \frac{5}{4} \Rightarrow x = -\frac{8}{5}, y = \frac{6}{5}
\]

\[
\frac{8/5, -6/5}{-8/5, 6/5} = 10 \quad \frac{8/5, -6/5}{-8/5, 6/5} = -10.
\]