SOME HINTS ON HOMEWORK 5

Chapter 5

1. The lefthand side is functions from U to functions from T to S. The righthand side is functions from $T \times U$ to S. Find a way to convert one type of function to the other.

4. The set $\{a\}$ is an element of $\{\{a\}\}$. And so forth.

7. Use the lexicographic ordering. This is explained in problem 11 in Chapter 4.

8. The Axiom of Regularity rules out a set being an element of itself. Every set is a subset of itself.

17. With shoes you can choose the left shoe from each pair. With socks you cannot differentiate left from right.

18. Prove the contrapositive.

22. A set and its power set always have different numbers of elements. So this cannot happen.

25. The rationals and the reals are not well ordered.

27. The reals and the rationals are two examples.

Chapter 6

1. The rationals are closed under all arithmetic operations, as long as we do not divide by 0. The irrationals are not closed under any arithmetic operations. For instance $\sqrt{3} \cdot \sqrt{3} = 3$, so the product of two irrationals is rational.

3. For each equivalence class you can find a representative of the form $(k, 0)$ with $k \in \mathbb{Z}$. Conversely, if $k \in \mathbb{Z}$, then $[(k, 0)]$ is an equivalence class in your construction. So the process here just recreates the integers.

4. Let $x_j = q + \sqrt{2}/j$.

5. Let $x = \pi + e$ and $y = \pi - e$. If both were rational then $x + y$ would be rational so 2π is rational, which is false.

6. The set must be an interval.

9. Imitate closely the proof in the text that addition is well defined. Of course use the definition of subtraction given in the text. The one change in the proof is that, when you add the two equations together, do *not* add left to left and right to right. Instead add left to right and right to left. Then it all works out.