SOME HINTS ON HOMEWORK 6

Chapter 6

11. As you know, we cannot divide 1 by 5 in the integers. Now $1 = (1, 2)$ and $5 = (1, 6)$. If there were an integer $n = (p, q)$ such that $5 \cdot n = 1$ then this would give the equation

$$(1, 6) \cdot (p, q) = (1, 2)\,.$$

Write this out and it leads to a contradiction.

15. If the number is rational then

$$\sqrt{2} + \sqrt{3} = \frac{p}{q}\,,$$

where p, q are integers in lowest terms. Now square both sides and manipulate. You will derive that $\sqrt{6}$ is rational, and that is false.

17. The complex numbers are not defined by equivalence classes.

23. Set

$$(re^{i\theta})^3 = 1 = 1 \cdot e^{i0}$$

and solve. Then set

$$(re^{i\theta})^3 = 1 = 1 \cdot e^{i2\pi}$$

and solve. Finally set

$$(re^{i\theta})^3 = 1 = 1 \cdot e^{i4\pi}$$

and solve.

27. Use Euler's formula and invoke the fact that sine and cosine are 2π-periodic.

29. Similar to problem 23 above.

39. Just write it out, using the definitions of addition and multiplication.

40. Similar to number 39.

43. If you perform the multiplication you end up with

$$(z_1^2 + z_2^2 + z_3^2 + z_4^2)\mathbf{1}\,.$$

45. Use the Euclidean algorithm to write

$$p(z) = (z - \alpha) \cdot q(z) + r(z)\,,$$

where the degree of r is less than the degree of $(z - \alpha)$. So r is a constant. Substitute in $z = \alpha$ to see that in fact r must be 0.