SOME HINTS ON HOMEWORK 7

Chapter 7

1. At the first step remove an interval of length $\rho > 0$. At the second step remove two intervals of length ρ^2. At the third step remove four intervals of length ρ^3 and so forth. Add up the lengths of all the removed intervals. Set that answer equal to η and solve for ρ.

2. The complement of the Cantor set is dense in the unit interval. That is to say, it intersects every nontrivial sub-interval.

3. There are countably many removed intervals and each has two endpoint.

9. If q is a rational number and $I = (q - \epsilon, q + \epsilon)$ is a small interval centered at q, then I will contain infinitely many rationals. So the set of rationals is NOT discrete. Every point of the Cantor set is a limit point of the Cantor set, so the Cantor set is not discrete. The set of integers is discrete. If n is an integer, then $(n - 1/2, n + 1/2)$ contains no other integers. The set T is certainly discrete.

11. If the set of s_j is bounded above, then let α be the least upper bound of the set. Then $s_j \to \alpha$. Otherwise the set is unbounded.

16. Define a relation on U by $x \sim y$ if all numbers between x and y lie in U. This is an equivalence relation. The equivalence classes are the intervals that we seek.

19. Let $T = \{t_j\}$ be an enumeration of the rationals. Then the closure X of T is the entire real line. So $X \setminus T$ is uncountable.

21. The union is certainly open. If instead $\mathcal{O}_j = (-1/j, 1/j)$, then each $\mathcal{O}_j$ is open but $\cap_j \mathcal{O}_j = \{0\}$ which is closed and not open.

Define $\mathcal{E}_j = [1/j, 1 - 1/j]$. Then each $\mathcal{E}_j$ is closed, but the union is the open interval $(0, 1)$.

Chapter 8

2. Any union of sets of irrationals is also a set of irrationals. Any finite intersection of such sets is also such a set.

4. These do not form a topology because

$$\bigcup_j \{j\}$$

is not a finite set. So not closed under union.

17. $f : (-1, 1) \to (-1, 1)$ be given by $f(x) = 1-x^2$. Then $f^{-1}([-1/2, 1/2]) = (-1, -1/\sqrt{2}] \cup [1/\sqrt{2}, 1)$, which is *not* compact..

21. The closure of S is S itself. The interior of S is empty. The boundary of S is S.