SOME HINTS ON HOMEWORK 8

Chapter 8

30. Let E_1, E_2, $\ldots$, E_k be compact. If $\mathcal{U} = \{U_\alpha\}$ is an open covering of $\cup_j E_j$, then choose a finite subcovering $\mathcal{U}_1$ for E_1, choose a finite subcovering $\mathcal{U}_2$ for E_2, $\ldots$ choose a finite subcovering $\mathcal{U}_k$ for E_k. Then $\mathcal{V} = \cup_{j=1}^k \mathcal{U}_j$ is a finite subcovering for the union of the E_j.

A finite intersection is the same idea but even easier.

31. The finite subcover consists of $(-1/4, 1/4)$, $(5/6, 7/6)$, and the interval $(1/10, 9/10)$.

33. Let

$$f(x) = \begin{cases} 0 & \text{if} \quad x \leq 1 \\ (1/2)(x-1) & \text{if} \quad 1 < x < 3 \\ 1 & \text{if} \quad 3 \leq x\,. \end{cases}$$

Chapter 11

1. Define $(g,h)\cdot(g',h') = (g\cdot g', h\cdot h')$. If G and H are abelian then $G\times H$ will also be abelian.

2. The group G has only one element of order 2, and that is 2. Namely $2+2=4$. But H has at least two elements of order 2, namely $(2,0)$ and $(0,2)$. So the groups cannot be isomorphic.

4. Call the group elements X, Y, Z. Then we can define a group by

$$X\cdot X = X\,,$$

$$X\cdot Y = Y\cdot X = Y\,,$$

$$X \cdot Z = Z \cdot X = Z\,,$$
$$Y \cdot Z = Z \cdot Y = X\,.$$

This is the same as $\mathbb{Z}_3$. Any other three-element group that you cook up will be isomorphic to this one.

10. S is closed under addition. The first T is not. S is closed under multiplication, and so is the first T.

The second T is closed under addition. It is also closed under multiplication.

12. The integers $\mathbb{Z}$ equipped with addition is an infinite cyclic group. The group $\mathbb{Z}_3$ with addition is a finite cyclic group.

23. Let $k = 2$. We see that

$$\begin{aligned}
[\phi(g)]^2 &= \phi(g) \cdot \phi(g) \\
&= \phi(g \cdot g) \\
&= \phi(e_G) \\
&= e_H\,.
\end{aligned}$$

The general case follows by induction.