PRACTICE EXAM FOR SECOND MIDTERM

(8 points) 1. Let $S = \{3, 4, 6\}$ and $T = \{3, 5\}$. What is $S \cup T$? What is $S \cap T$? What is $S \setminus T$?
2. Let $S = \{2, 4\}$ and $T = \{a, b, d\}$. What is $S \times T$? What is $T \times S$?

3. Draw two Venn diagrams to illustrate the identity

$$(T \cup U) \setminus S = (T \setminus S) \cup (U \setminus S).$$
4. What is the power set of \{\lambda, A, 2\}?
(12 points) 5. Which of these functions is one-to-one? Which is onto (give a brief reason for each answer)?

(a) \( f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2 + x \)

(b) \( g : \mathbb{N} \rightarrow \mathbb{N} \quad g(n) = n(n + 3) \)

(c) \( h : \mathbb{R} \rightarrow \mathbb{R} \quad h(x) = x \cos x \)

(12 points) 6. Which of these sets is countable and which uncountable (give a brief reason for each answer)?
(a) $\mathbb{C} \times \mathbb{C}$

(b) $\mathbb{Z} \times \mathbb{Q}$

(c) $\mathbb{N} \times \mathbb{R}$

(10 points) 7. Calculate the inverse of the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ x & \text{if } x > 0. \end{cases}$$
(14 points) 8. Prove that the collection of $S$ of rational numbers with denominator 7 is countable.
9. Explain why the product of an uncountable set and an uncountable set is uncountable.

10. Explain why the union of a uncountable set and an uncountable set is uncountable.