Project:

Math 318 HW7 Solutions

Notes:

Section 4.1

3 b) $\begin{bmatrix} 2 & -2 & 4 \\ -1 & 1 & -2 \\ 3 & -3 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 3 & -3 & 6 \end{bmatrix}$

$\to \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 3 & -3 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 - x_2 + 2x_3 = 0$

This is a plane.

d) $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 2 & -4 & 3 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$\to \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$x_4 = t$

$x_2 = s$

$x_3 = x_4 = t$

$x_1 = 2x_2 - x_4 = 2s - t$

So sol. is

$s\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ This is a plane.

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f) $\begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ -1 & -3 & 1 & 2 & 3 \\ 1 & -1 & 3 & 1 & 1 \\ 2 & -3 & 7 & 3 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 & 2 \\ 0 & -3 & 3 & 2 & 2 \\ 0 & -7 & 7 & 5 & 6 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 & 2 \\ 0 & 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & -2 & -8 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_5 = t$

$x_3 = s$

$x_4 = -4t$

$x_2 = s - 2t$

$x_1 = -2s + t$

$\underline{x} = s \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 0 \\ -4 \\ 1 \end{bmatrix}$ This is a plane,

To Do:

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4. b) $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 3 & 3 & 2 & 0 & 17 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 5 \\ 0 & 0 & +1 & +3 & +1 \end{array}\right]$

$x_4 = t$

$x_2 = s$

$x_3 = -3t + 1$

$x_1 = -s + t + 5$.

$$\underline{x} = s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

This is a plane.

To Do:

6 b) Need to find $\langle x_1, x_2, x_3, x_4 \rangle$ s.t.

$x_1 + x_2 + x_3 + x_4 = 0$

$2x_1 + 2x_2 + x_3 + 2x_4 = 0$

$2x_1 + 3x_2 + 2x_3 + 4x_4 = 0$

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$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 \\ 1 & 3 & 2 & 4 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

To Do:

$\therefore \quad x_4 = t$

$x_3 = -t$

$x_2 = -t$

$x_1 = t$

$\underset{\sim}{x} = t\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$. This is a line.

The vector we seek is $\langle 1, -1, -1, 1 \rangle$.

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b) $$c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$c_1 + 0c_2 + c_3 = 1$$
$$0c_1 - c_2 - 2c_3 = -1$$
$$1c_1 + 0c_2 + 1c_3 = 1$$
$$-2c_1 + 1c_2 + 0c_3 = -1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 1 & 0 & 1 & 1 \\ -2 & 1 & 0 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & -1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$c_3 = t$$
$$c_2 = -2t + 1$$
$$c_1 = -t + 1$$

$$\underline{x} = t \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$ This is a line.

Taking $t = 0$, we see that we may choose

$$c_1 = 1$$
$$c_2 = 1$$
$$c_3 = 0$$

To Do:

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12 b)

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & b_1 \\ 0 & 1 & -1 & b_2 \\ 1 & 1 & 1 & b_3 \\ 1 & 2 & 0 & b_4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & b_1 \\ 0 & 1 & -1 & b_2 \\ 0 & 1 & -1 & b_3 - b_1 \\ 0 & 2 & -2 & b_4 - b_1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & b_1 \\ 0 & 1 & -1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 - b_2 \\ 0 & 2 & -2 & b_4 - b_1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & b_1 \\ 0 & 1 & -1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 - b_2 \\ 0 & 0 & 0 & b_4 - b_1 - 2b_2 \end{array}\right]$$

Thus $b_4 - b_1 - 2b_2 = 0$

$b_3 - b_1 - b_2 = 0$.

This is the intersection of two hyperplanes, a two-dimensional set.

To Do:

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Notes:

Section 4.2

2 b) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

To Do:

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

Inverse is $\begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

Project:

Notes:

d) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -12 & -7 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -5/3 & 2/3 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & -1 & 1/2 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$

To Do:

On the lower left it's all 0s. So not invertible

3 b) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \end{array}\right]$

Project:

Notes:

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1/2 & -3 & 1/2 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1/2 & 1 & -1/2 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1/2 & -3 & 1/2 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 2 & 0 & 18 & -2 & -6 \\ 0 & 0 & 1 & -6 & 1 & 2 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 9 & -1 & -3 \\ 0 & 0 & 1 & -6 & 1 & 2 \end{array}\right]$$

So inverse is $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & -1 & -3 \\ -6 & 1 & 2 \end{bmatrix}$

$$A\underline{x} = \underline{b}$$

$$A^{-1}A\underline{x} = A^{-1}\underline{b}$$

$$\therefore \underline{x} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & -1 & -3 \\ -6 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

To Do:

$$c_1\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Project:

Notes:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & -1 & -3 \\ -6 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

So

$$0 \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

To Do:

d)

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

Project:

Notes:

$$\rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 4 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 4 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

$$\underline{x} = A^{-1}\underline{b} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

To Do:

$$2\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Project:

Notes: 4

a) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a-c+e & b-d+f \\ 2a-c & 2b-d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a - c + e = 1$

$b - d + f = 0$

$2a - c = 0$

$2b - d = 1$

One set of solutions is

$a = 1,\ b = 1,\ c = 2,\ d = 1,\ e = 2,\ f = 0$

Another set of solutions is

$a = 2,\ b = 2,\ c = 4,\ d = 3,\ e = 3,\ f = 1$

So the two right inverses are

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 2 & 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 2 \\ 4 & 3 \\ 3 & 1 \end{bmatrix}$.

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ has no right inverse,

To Do:

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Notes:

7 a) The inverse is

$$\left[\begin{array}{c|c} A^{-1} & 0 \\ \hline 0 & B^{-1} \end{array}\right].$$

To Do: