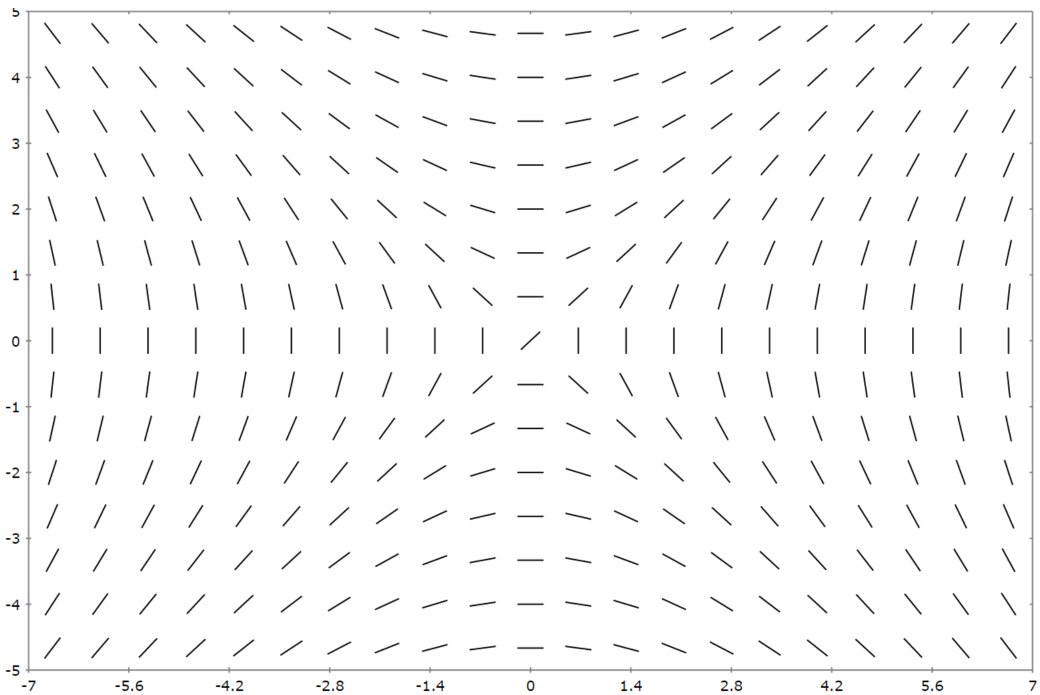
Math 217 Exam 1 Sept 17, 2015

Instructions:

1. There are three parts in this exam. Part I is multiple choice, Part II is True/False, and Part III consists of hand-graded problems.
2. The total number of points is 100.
3. You may use a calculator.
4. The scorecard and Part III will be collected at the end of the exam. You may take Part I and Part II with you at the end of the exam.

Part I. Multiple Choices $\quad 5 \times 10 = 50$ points

1. The direction field below corresponds which differential equation?



A. $y' = \frac{x}{y}$

B. $y' = x^2 + y^2$

C. $y' = -\frac{x}{y}$

D. $y' = x + y$

E. $y' = x - y$

F. $y' = x^2 - y^2$

2. Classify the differential equation $(\sin t)y'' + ty^2 = e^t$.

A. ordinary, linear, order 1

B. ordinary, linear, order 2

C. ordinary, non-linear, order 1

D. ordinary, non-linear, order 2

E. partial, linear, order 1

F. partial, non-linear, order 2

3. Which of the following is a solution of the differential equation $y' = y + 2e^{-t}$?

A. $y(t) = e^{-t}$

B. $y(t) = \sin t$

C. $y(t) = e^{t} + e^{-t}$

D. $y(t) = e^{t} - e^{-t}$

E. $y(t) = \cos t$

F. none of the above

4. The differential equation

$$\frac{xy'}{y} + e^y y' + \cos x + \ln y = 0$$

belongs to only one of the following categories. Which one is it?

A. 1st order linear

B. separable

C. exact

D. 2nd order linear

E. 2nd order non-linear

F. autonomous

5. Solve the initial value problem

$$y' + 3t^2 y = 0, \quad y(0) = 7.$$

A. $y(t) = e^{-t}$

B. $y(t) = 7e^{-t}$

C. $y(t) = \frac{7}{2}(e^{-t^2} + e^{t^2})$

D. $y(t) = 7e^{3t^2}$

E. $y(t) = 7e^{-t^3}$

F. none of the above

6. If ϕ is a solution of $y' = 6t(y-1)^{\frac{2}{3}}$ and $\phi(0) = 1$, what is $\phi(1)$?

A. 0

B. 1

C. 2

D. 9

E. 33

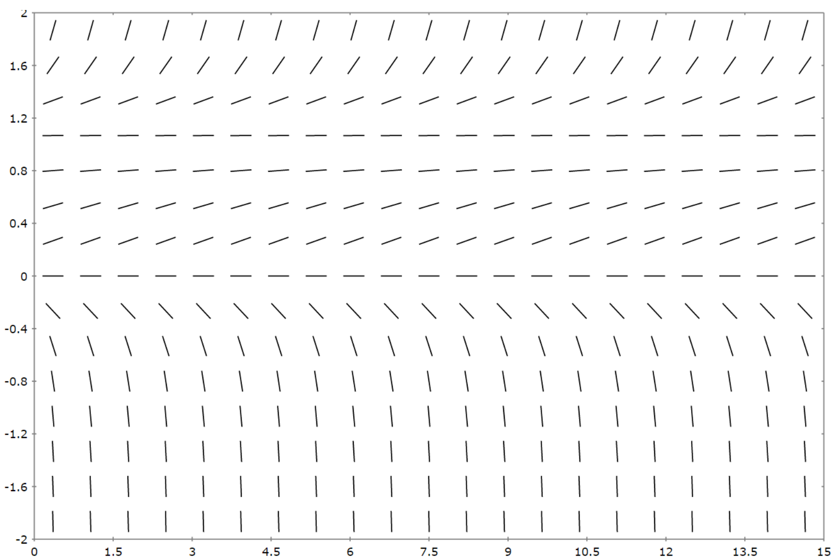
F. none of the above

7. Determine the interval in whih the solution exists for the initial value problem

$$y' + y^3 = 0, \quad y(0) = 1.$$

A. $(-\infty, -\frac{1}{2})$
B. $(-\frac{1}{2}, \infty)$
C. $(-\infty, 0)$
D. $(0, \infty)$
E. $(-\frac{1}{2}, \frac{1}{2})$
F. none of the above

8. Only one of the statements is true for the direction field below. Which one is it?



A. $y(t) = 0$ is a stable equilibrium solution.

B. $y(t) = 0$ is an unstable equilibrium solution.

C. $y(t) = 1$ is a stable equilibrium solution.

D. $y(t) = 1$ is an unstable equilibrium solution.

E. Neither $y(t) = 0$ nor $y(t) = 1$ are equilibrium solutions.

F. None of the above.

9. Find an integrating factor μ that will make the following equation exact.

$$y + 2tyy' = e^{-2y}y'$$

A. $\mu = e^{2y - \ln y}$

B. $\mu = \frac{1-2y}{2ty - e^{-2y}}$

C. $\mu = e^{2t - \ln t}$

D. $\mu = e^{\frac{1}{2t}}$

E. $\mu = e^{-\frac{1}{2y}}$

F. None of the above.

10. Which of the initial value problem below DOES NOT have a unique solution?

A. $y' = 0, \qquad y(0) = 0$

B. $y' = y^{\frac{1}{2}}, \qquad y(0) = 0$

C. $y' = y, \qquad y(0) = 0$

D. $y' = y^2, \qquad y(0) = 0$

E. $y' = y^{2015}, \qquad y(0) = 0$

F. None of the above.

Part II. True/False $5 \times 2 = 10$ points

Choose 'A' if the statement is true; choose 'B' if the statement is false.

11. Only 1st order differential equations can be solved.

12. The function $y(t) = -2t$ is an equilibrium solution to the differential equation $y' = y + 2t$.

13. Consider the autonomous equation $\frac{dy}{dt} = f(y)$, where f is a continuous function. It is NOT possible to have two stable equilibrium solutions with no other equilibrium solution between them.

14. The function $\mu(t) = e^{2t}$ is an integrating factor for the equation

$$ty' + 2y + e^t = 0.$$

15. There is only one solution to the initial value problem:

$$y' = 5t^2, \qquad y(1) = 2\pi.$$

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Part III will be collected separately. Please write down your name and your student number.
Student No.

Name:

For graders:

16.

17.

18a.

18b.

Total:

Part III. Hand-graded problems $\qquad 10 + 10 + 20 = 40$ points

16. (10 points)
Solve the initial value problem below.

$$t\frac{dy}{dt} + 2y = \sin t, \qquad y\left(\frac{\pi}{2}\right) = 0.$$

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17. (10 points)
Solve the differential equation below.

$$\frac{dy}{dx} = \frac{x^2 - y^2}{x^2 - xy}$$

Hint: Try the substitution $v = \frac{y}{x}$.

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18. (10 + 10 = 20 points)

The radiation of a black body is governed by the the Stefan-Boltzmann law. From it, we could derive a model for the variation of the temperature of a body with respect to its surroundings. The model is described by the differential equation

$$\frac{du}{dt} = -\alpha \left(u^4 - T^4 \right)$$

where $u(t)$ is the temperature of the body at time t measured in Kelvin, T is the ambient temperature which we keep constant, and α is an constant.

(a) Solve the differential equation.

Note: An implicit solution is good enough. You need to do a rather difficult integration.

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(b) Now assume that the object is in vacumm, i.e. $T = 0K$. Then the model is simplified to be

$$\frac{du}{dt} = -\alpha u^4$$

Let us say $\alpha = 2.0 \times 10^{-10} K^{-3}/s$, the initial temperature $u(0) = 300K$. Calculate the time for the temperature of the body to reach $150K$. (Round it to the nearest second.)

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