Math 217 Exam 2 Oct 20, 2015

Instructions:

1. There are three parts in this exam. Part I is multiple choice, Part II is True/False, and Part III consists of hand-graded problems.
2. The total number of points is 100.
3. You may use a calculator.
4. The scantron and Part III will be collected at the end of the exam. You may take Part I and Part II with you at the end of the exam.

Here are some definitions/identities that might be useful:

$$\cosh t = \frac{e^t + e^{-t}}{2}$$
$$\sinh t = \frac{e^t - e^{-t}}{2}$$
$$e^{it} = \cos t + i \sin t$$
$$\cos t = \frac{e^{it} + e^{-it}}{2}$$
$$\sin t = \frac{e^{it} - e^{-it}}{2i}$$
$$\int \sec t dt = \ln|\sec t + \tan t| + C$$
$$\int \csc t dt = \ln|\csc t - \cot t| + C$$

Part I. Multiple Choices $\quad 5 \times 10 = 50$ points

1. Consider the 2nd order linear equation with constant coefficients:

$$y'' + ay' + by = 0.$$

If r_1 and r_2 are the roots of its characterisitic equation, then what is $r_1^2 + r_2^2$?

A. 1
B. $\sqrt{a^2 - 4b}$
C. $a^2 - 4b$
D. $a^2 + 2b$
E. $a^2 - 2b$
F. none of the above

2. Find the largest interval on which the following IVP has a unique solution.

$$(2t+1)y'' + \sin t \cdot y' + e^{t-3}y = \tan t, \qquad y(0) = 0, \quad y'(0) = 1.$$

A. $[0, \infty)$
B. $(-\frac{1}{2}, \infty)$
C. $(-\frac{\pi}{2}, \infty)$
D. $(-\frac{1}{2}, \frac{\pi}{2})$
E. $(-\pi, \pi)$
F. none of the above

3. Consider the differential equation

$$y'' - y = 0. \tag{1}$$

Which of the following is NOT a solution of (1)?

A. 0
B. e^t
C. e^{-t}
D. $\cosh t$
E. $\cosh(t + 1)$
F. all of the above are solutions of (1)

4. Consider the differential equation

$$y'' + y = 0. \tag{2}$$

Which of the following is NOT a solution of (2)?

A. 0
B. $\sin t$
C. $\cos t$
D. $\cos(t+1)$
E. $\tan t$
F. all of the above are solutions of (2)

5. Find the general solution of

$$2y'' - 7y' + 3y = 0.$$

A. $y(t) = c_1 e^{-\frac{1}{2}t} + c_2 e^{3t}$

B. $y(t) = c_1 e^{-\frac{1}{2}t} + c_2 e^{-3t}$

C. $y(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{3t}$

D. $y(t) = c_1 e^{\frac{1}{2}t} + c_2$

E. $y(t) = c_1 + c_2 e^{-3t}$

F. none of the above

6. Find the solution to the IVP below:

$$y'' + 2y' + y = 0, \qquad y(0) = 5, \quad y'(0) = -3.$$

A. $5e^{-t} - 2te^{-t}$

B. $5e^{-t} + 2te^{-t}$

C. $5e^{-t} + 3e^{t}$

D. $5e^{-t} - 3te^{-t}$

E. $5e^{-t} - 2e^{t}$

F. none of the above

7. Compute the Wronskian $W(y_1, y_2)(t)$ for $t > 0$ where

$$y_1(t) = t \ln t, \qquad y_2(t) = t^2.$$

A. $t^2(\ln t + 1)$
B. $t^2(\ln t - 1)$
C. $t^2 \ln t$
D. $t \ln t + t^2$
E. 0
F. none of the above

8. Consider the differential equation

$$3y'' + y' - 2y = 2\cos t \tag{3}$$

Which of the following is the general solution of (3)?

A. $c_1 e^{\frac{2}{3}t} + c_2 e^{-t} - \frac{5}{13}\cos t + \frac{1}{13}\sin t$
B. $c_1 e^{\frac{2}{3}t} + c_2 e^{-t} - \frac{5}{13}\cosh t + \frac{1}{13}\sinh t$
C. $c_1 e^{\frac{2}{3}t} + c_2 e^{t} + \frac{5}{26}e^{it} + \frac{1}{26}e^{-it}$
D. $c_1 e^{\frac{2}{3}t} + c_2 e^{t} - \frac{5}{13}\cos t + \frac{1}{13}\sin t$
E. $c_1 e^{\frac{2}{3}t} + c_2 e^{t} - \frac{5}{13}e^{t} + \frac{1}{13}e^{-t}$
F. none of the above

9. Consider the differential equation

$$t^2 y'' - t(t+2)y' + (t+2)y = 0, \qquad t > 0. \tag{4}$$

The function $y_1(t) = t$ is a solution of (4). Choose the function y_2 such that y_1 and y_2 form a fundamental set of solutions to (4).

A. $y_2(t) = t^2$

B. $y_2(t) = e^t$

C. $y_2(t) = te^t$

D. $y_2(t) = t^2 e^t$

E. $y_2(t) = \ln t$

F. none of the above

10. Consider the differential equation

$$y''' + y'' = 3e^t + 4t^2 \tag{5}$$

Which of the following is NOT a solution of (5)?

A. $\frac{3}{2}e^t + 4t^2 - \frac{4}{3}t^3 + \frac{1}{3}t^4$
B. $\frac{3}{2}e^t + 1 + 4t^2 - \frac{4}{3}t^3 + \frac{1}{3}t^4$
C. $\frac{3}{2}e^t + 1 + t + 4t^2 - \frac{4}{3}t^3 + \frac{1}{3}t^4$
D. $\frac{3}{2}e^t + 2015t + 4t^2 - \frac{4}{3}t^3 + \frac{1}{3}t^4$
E. $2\cosh t + 1 + t + 4t^2 - \frac{4}{3}t^3 + \frac{1}{3}t^4$
F. all of the above are solutions of (5)

Part II. True/False $5 \times 2 = 10$ points

Choose 'A' if the statement is true; choose 'B' if the statement is false.

11. For a second order linear homogeneous ordinary differential equation with constant coefficients, if the characteristic equation has no real roots, then we cannot solve the equation.

12. Let p and q be continuous functions, and let y_1 and y_2 be solutions of the differential equation

$$y'' + p(t)y' + q(t)y = 0.$$

If $W(y_1, y_2)(0) \neq 0$, then $W(y_1, y_2)(t) \neq 0$ for all t.

13. The functions $y_1, y_2, \ldots, y_n$ are linearly independent, if and only if

$$W(y_1, y_2, \ldots, y_n)(t_0) \neq 0$$

for some t_0.

14. The quasi-frenquency of the dampened spring is always smaller than the natural frequency.

15. There are continuous functions p and q such that

$$y_1(t) = t \ln t, \qquad y_2(t) = t^2$$

form a fundamental set of solutions of the second order homogeneous differential equation

$$y'' + p(t)y' + q(t)y = 0, \qquad t > 0.$$

Hint: You may use your result in 7.

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Part III will be collected separately. Please write down your name and your student number.
Student No.

Name:

For graders:

16.

17.

18

19.

Total:

Part III. Hand-graded problems $10 + 10 + 20 = 40$ points

16. (5+5 = 10 points)
Let m, γ and k be positive constant. Find the general solution of equation

$$mu''(t) + \gamma u'(t) + ku(t) = 0$$

in each of the two cases below:

(a) $\gamma^2 - 4km < 0$

(b) $\gamma^2 - 4km > 0$

17. (10 points)
Find the general solution of the equation

$$t^2 y'' + 2ty' - 12y = 0, \qquad t > 0$$

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18. (10 points)
Find the general solution of the equation

$$y^{(3)} + y' - 10y = 0.$$

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19. (10 points)
Find the general solution of the equation

$$y'' + y = \tan t.$$

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