

Ma 322: Biostatistics

Homework Assignment 5

Prof. Wickerhauser

Due Friday, February 24th, 2012

Read Chapter 10, “Stochastic Processes and Markov Chains,” pages 160–184 of our text.

Note: Although our text has no index or table of contents, it is easy to locate words in the electronic version using the Find function of your favorite PDF reader.

1. Suppose that the list of variables $p = (p_1, \dots, p_K)$ has a Dirichlet prior density

$$f_\alpha(p) \propto p_1^{\alpha_1-1} \cdots p_K^{\alpha_K-1},$$

where $\alpha = (\alpha_1, \dots, \alpha_K)$ is a list of shape parameters. We perform an experiment that yields counts $n = (n_1, \dots, n_K)$ having the multinomial likelihood

$$L_n(p) \propto p_1^{n_1} \cdots p_K^{n_K}.$$

- (a) For what values of α does one get a non-informative Dirichlet prior pdf?
 - (b) Determine the shape parameters for the posterior pdf $L_n(p)f_\alpha(p)$.
2. Implement the function `Walk1d()` on p.167 of our text and graph three 100-step simulations starting from random seeds 123 and 4567 and 89012.
 3. Implement the function `Walk2d()` on p.168 of our text and graph three 500-step simulations starting from random seeds 123 and 4567 and 89012.
 4. A restless koala moves among three eucalyptus trees, which are labeled 1, 2, and 3. A patient park ranger watches and makes notes whenever the koala changes trees, producing the following table:

Koala Tree-Change Counts

Start Tree	Finish Tree	Count
1	2	12
1	3	8
2	1	5
2	3	20
3	1	3
3	2	7

- (a) Treat the koala’s movements as a Markov process and determine the transition matrix M from this table of counts.
- (b) Starting with a uniform prior distribution on the three trees and assuming the koala’s tree-change preferences remain the same, compute the posterior koala distribution, namely the stationary distribution determined by M .
- (c) Find the stationary transition matrix for this koala.

5. Consider the following transition matrix for a 4-state Markov chain:

$$M = \begin{pmatrix} 0.2 & 0.1 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \end{pmatrix}.$$

- (a) Is M periodic or aperiodic?
- (b) Is M irreducible?
- (c) Is M ergodic?
- (d) Does M have a stationary distribution?
- (e) Is M reversible?

6. Consider the following transition matrix for a 5-state Markov chain:

$$F = \begin{pmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \\ 0.0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.0 & 0.0 & 0.33 & 0.33 & 0.34 \\ 0.0 & 0.0 & 0.0 & 0.50 & 0.50 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.00 \end{pmatrix}.$$

- (a) Is F periodic or aperiodic?
- (b) Is F irreducible?
- (c) Is F ergodic?
- (d) Does F have a stationary distribution?
- (e) Is F reversible?