1. Let \( f_n(x) = [x^n(1 - x^n)] \) for \( n = 1, 2, 3, \ldots \). Does the sequence \( \{f_n(x)\} \) converge uniformly on \( 0 < x < 1 \)?

2. Use Cauchy’s Inequalities to deduce Liouville’s Theorem.

3. Let \( D \subset \mathbb{C} \) be the closed diamond-shaped region with vertices \( 1, i, -1, -i \). Suppose that \( f = f(z) \) is analytic on \( D \) and satisfies \( |f(z)| \leq M \) for all \( z \in D \). Prove that \( |f'(0)| \leq M\sqrt{2} \) and \( |f''(0)| \leq 4M \).

4. Suppose that \( f(z) \) is analytic on \( |z| < 2 \). Define \( F_0(z) = f(z) \) and \( F_{n+1}(z) = \int_0^z F_n(w) \, dw \) for \( n \geq 0 \). Prove that if \( \{F_n(z)\} \) converges uniformly on \( |z| < 1 \), then \( f(z) = ce^z \) for some constant \( c \).

5. Recall that \( \lim_{n \to \infty} \left(1 + \frac{z}{n}\right)^n = e^z \) for all real \( x \). Show that

\[
\lim_{n \to \infty} \left(1 + \frac{z}{n}\right)^n = e^z,
\]

for all complex \( z \). (Hint: use the uniform convergence theorem and the coincidence principle.)

6. Compute \( \Gamma(3/2) \) and \( \Gamma(-1/2) \).