Ma 416: Complex Variables
Homework Assignment 7

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Due Thursday, October 27, 2005


1. Use the argument principle to count the zeros of \( P(z) = z^4 + z^3 + 6z^2 + 3z + 5 \) in the left half-plane \( \{ \Re z < 0 \} \) and right half-plane \( \{ \Re z > 0 \} \) of the complex plane.

2. Use Rouché's theorem to determine the number of zeros of \( 3e^{z/2} + z \) satisfying \( |z| < 1 \).

3. Suppose \( \{ f_n : n = 1, 2, \ldots \} \) is an infinite sequence of analytic functions that converges uniformly in all compact subsets of a region \( D \) containing 0.
   (a) Show that \( \{ \exp(f_n) : n = 1, 2, \ldots \} \) is also an infinite sequence of analytic functions that converges uniformly in each compact subset of \( D \).
   (b) Show that if \( \lim_{n \to \infty} \exp(f_n(0)) = 0 \), then \( \lim_{n \to \infty} \exp(f_n(z)) = 0 \) for all \( z \in D \).

4. Is it possible for a function \( f = f(z) \) which takes only purely imaginary values to be analytic on \( \{|z| < 1\} \)?

5. Show that \( f(z) = z/(1 - z)^2 \) is univalent in \( |z| < 1 \).

6. Prove that the converse to Darboux's theorem is false: Find a simple closed curve \( S \) and an analytic function \( f = f(z) \) such that \( f \) is univalent inside \( S \) but not univalent on \( S \).