Ma 449: Numerical Applied Mathematics — Final Examination.

Prof. Wickerhauser

Due in Room 100, Cupples I Hall
by **3:00pm Friday, December 16th, 2011**

5 problems on 1 page

You may use a calculator or computer and refer to the textbook, your class notes, and the homework sets and model solutions on the Math 449 web site. No collaboration with any other person is allowed.

1. Suppose $f = f(x)$ has continuous derivatives of all orders satisfying $|d^k f(x)/dx^k| \leq M^k$ for all x and all $k = 1, 2, 3, \ldots$, where M is some positive constant.

   Use Taylor's theorem to estimate the error in the difference formula $f''(x) \approx [f(x+h) - 2f(x) + f(x-h)]/h^2$, in terms of M, as $h \to 0$.

2. Use the composite trapezoid rule with Richardson extrapolation to compute the following integral to 13 significant digits:

$$\int_{-1}^{2} \exp(t^2 \cos t)\, dt$$

3. The function $f(x, y) = 5e^{2x} - 4e^x + 8ye^x + 4y^2 + 17$ has a unique minimum in the x, y plane.

   (a) Find the minimum (x, y) to 13 significant digits in both x and y. Suggestion: use gradients.

   (b) Starting with the initial simplex $(0, 0)$, $(0, 1)$, $(1, 0)$, perform one step of the Nelder-Mead algorithm to find the next approximating simplex.

4. Consider the following initial value problem on the interval $[0, 1]$:

$$y'(t) = (2 - t^2)y(t), \quad 0 < t < 1; \qquad y(0) = 1.$$

   Use the fourth-order Runge-Kutta method (RK4) with a step size small enough to give a 13 significant digit approximation to $y(1)$.

5. Consider the following boundary value problem on the interval $[0, 1]$:

$$x''(t) = (2 - t^2)x(t); \qquad x(0) = 1, \quad x(1) = 1,$$

   Find approximate solutions for $x(0.5)$ by each of the following methods:

   (a) the finite differences method with step size $h = 0.02$;

   (a') the finite differences method with step size $h = 0.01$;

   (b) linear shooting with RK4 and step size $h = 0.02$.

   (b') linear shooting with RK4 and step size $h = 0.01$.

   Finally, combine the results from a,a' and b,b' to compute a best estimate of $x(0.5)$ for each method. Justify your claim for the number of significant digits in each best approximation.