

**Math 449: Numerical Applied Mathematics  
Midterm Examination**

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You may use a calculator, the textbook, your class notes, and the model homework solutions published this semester. Please write your answers in the bluebook.

**Problem 1.** Express 1.01011 01111 110 (base 2) in base 10 notation, giving at least four significant digits.

**Problem 2.** (a) Find a polynomial  $p = p(h)$  of minimal degree in  $h$  such that  $\ln(1 + h) = p(h) + O(h^3)$  as  $h \rightarrow 0$ . (b) Find  $\epsilon > 0$  such that  $|\ln(1 + h) - p(h)| < 0.0005$  whenever  $|h| < \epsilon$ .

**Problem 3.** (a) Solve the equation  $9x - \cos x = 4$  for  $x$  to four significant digits. (b) Prove that your method gives the required accuracy.

**Problem 4.** Let  $\mathbf{x} = (1, -1, 2)$  and  $\mathbf{y}$  be two vectors in  $\mathbf{R}^3$ . Suppose  $\mathbf{y} \cdot \mathbf{y} = 6$  and  $\mathbf{x} \cdot (2\mathbf{y}) = 3$ . Compute the secant of the angle between  $\mathbf{x}$  and  $\mathbf{y}$ .

For the following three problems, let

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

**Problem 5.** Find a factorization  $A = LU$ , where  $L$  is unit lower triangular and  $U$  is upper triangular. (Hint: no interchanges are needed.)

**Problem 6.** Find the determinant  $\det A$ .

**Problem 7.** Solve the linear system  $Ax = b$  for  $x$ , given  $b = (0, 0, 0, 0)$ .

**Problem 8.** Find the complex exponential Fourier series for the function  $f(x) = \cos^2(x)$ . (Hint: it has finitely many nonzero terms.)

**Problem 9.** Find the quadratic Lagrange polynomial that interpolates  $(1, 1)$ ,  $(2, 2)$ , and  $(3, 5)$ . Write it in Newton's form and also as  $ax^2 + bx + c$ .

**Problem 10.** Find the least squares curve of the form  $y(x) = Ax + B$  for the points  $(1, 1)$ ,  $(2, 2)$ , and  $(3, 5)$ .