Math 449: Numerical Applied Mathematics
Midterm Examination

Prof. Wickerhauser
22 October 2010

You may use a calculator and the textbook. Please write your answers in the bluebook.

Problem 1. Express 1.01001 00001 (base 2) in base 10 notation, giving at least four significant digits.

Problem 2.
(a) Find a polynomial $p = p(h)$ of minimal degree in h such that $e^h = p(h) + O(h^3)$ as $h \to 0$.
(b) Find $\epsilon > 0$ such that $|e^h - p(h)| < 0.0005$ whenever $|h| < \epsilon$.

Problem 3. The function $f(x) = x^{1/3}$ has a unique root at 0.
(a) Find the Newton–Raphson iteration formula for the equation $f(x) = 0$.
(b) Prove that the iteration formula from part a does not converge from any initial point.

Problem 4. Let $\mathbf{x} = (1, -2, 2)$ and $\mathbf{y}$ be two vectors in $\mathbf{R}^3$. Suppose $\mathbf{y} \cdot \mathbf{y} = 4$ and $\mathbf{x} \cdot (3\mathbf{y}) = 2$. Compute the cosine of the angle between $\mathbf{x}$ and $\mathbf{y}$.

For the following three problems, let

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

Problem 5. Find a factorization $A = LU$, where L is unit lower triangular and U is upper triangular. (Hint: no interchanges are needed.)

Problem 6. Find the determinant $\det A$.

Problem 7. Using your choice of matrix norms $\|\cdot\|_\infty$ or $\|\cdot\|_1$, compute the condition number of the matrix A.

Problem 8. Find the complex exponential Fourier series for the function $f(x) = \cos(x) + \sin(x)$. (Hint: it has finitely many nonzero terms.)

Problem 9. Fix $-1 < c < 1$ and let $P_0 = (-1, 0)$, $P_1 = (c, 1)$, and $P_2 = (1, 0)$ be three points in the (x, y)-plane. Let $B = B(t)$, $0 \le t \le 1$, be the Bézier curve with control points P_0, P_1, P_2. Find the maximum y-coordinate of $B(t)$ for any t and any c.

Problem 10. Find the least squares curve of the form $y(x) = Ax + B$ for the three points $(0, 0)$, $(1, 1)$, and $(2, 1)$.