Ma 450: Mathematics for Multimedia
Homework Assignment 1

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Due Friday, February 3rd, 2012

1. Suppose a divides b and b divides c and c divides d. Must a divide d?
2. Write a computer program that finds the greatest common divisor of three positive integers a, b, c, assuming that the greatest common divisor function `gcd(x,y)` for any two positive integers `x,y` has already been implemented.
3. Suppose that $a + b$ and $a - b$ are relatively prime. Must a and b be relatively prime?
4. Find the greatest common divisor of the two numbers 123 456 789 and 12 345 678 901 234 567 890.
5. Is there an integer x such that $3702x - 1$ is divisible by 85?
6. Suppose that $-2^{30} < x < 0$ and $-2^{30} < y < 0$ for integers x, y. Is it possible that the operation $x + y$ causes integer underflow on a 32-bit twos complement computer?
7. Express the integer 1011 1010 1100 (base 10) in hexadecimal.
8. Prove that if p is a prime number, then $\sqrt[3]{p}$ is not a rational number.
9. Write a computer program to read a string of up to six characters from the set $\{0, 1, \ldots, 9, a, b, \ldots, z\}$, considered as digits of a positive integer base 36, and then print the number's decimal digits.
10. Find the rational number represented by the repeating hexadecimal expansion $0.\overline{CAFE}$ (base 16).
11. What is the smallest positive subnormal number in IEEE double precision 64-bit binary floating-point format?
12. What will $\sum_{k=1}^{10^8} \frac{1}{k}$ equal on the example computer on p. 15, which uses IEEE 32-bit floating-point arithmetic? What if IEEE 64-bit floating-point arithmetic is used? What if the sum is computed in reverse order, starting with $k = 10^8$ and adding through $k = 1$?
13. Write a program to read 64-bit IEEE binary floating-point format and print the number in scientific notation. Have it treat NaN, $\pm\infty$, and ± 0 properly and have it signal when the number is subnormal.
14. Prove that the absolute error of a product of two numbers is not bounded by any fixed multiple of their summed absolute errors. Thus products behave badly with respect to absolute error.
15. Use Inequality 1.9 to prove that for any fixed $p > 0$, the computation $x \mapsto \sqrt[p]{x}$ is well-conditioned for all $x > 0$.