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MATH 2605

Quiz I

Given four points: $P_1(1,1,1)$, $P_2(2,-2,-1)$, $P_3(0,2,1)$ $P_4(1,0,1)$.

1. Write an equation in x, y, z for the plane (α) that passes through P_1, P_2, P_3
2. Do the four points lie on the same plane?
3. Write parametric equations for the line that passes through P_4 and is perpendicular to (α).

Solution:

1. Direction vectors: $\vec{v_1} = \overrightarrow{P_1P_2}\ (1,-3,-2)$
$\vec{v_2} = \overrightarrow{P_1P_3}\ (-1,1,0)$

Normal vector: $\vec{a} = \vec{v_1} \times \vec{v_2} = \left(\begin{vmatrix} -3 & 1 \\ -2 & 0 \end{vmatrix}, \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix}\right)$

$= (2, 2, -2)$

Equation: $\vec{a} \cdot ((x,y,z) - P_1) = 0$

$2x + 2y - 2z - 2 = 0$

$x + y - z - 1 = 0$

2. Since $1 + 0 - 1 - 1 = -1 \neq 0$, $P_4 \notin (\alpha)$. So the four points don't lie on the same plane

3. Since the line is perpendicular to (α), we can choose a direction vector to be $\vec{a}\ (2, 2, -2)$

parametric equations: $r(t) = (1,0,1) + t.(2,2,-2)$ or $\begin{cases} x(t) = 1 + 2t \\ y(t) = 2t \\ z(t) = 1 - 2t. \end{cases}$