Name:

Quiz # 3

09 | 16 | 2009

Section:

Given the curve:

$$r(t) = ti + 2tj + t^2k$$

Let $P(2,4,4)$ be a point on the curve.

1. Find the unit tangent vector to the curve at the point P.
2. Find the principal normal vector at P.
3. Write an equation in x, y, z for the osculating plane at P.

Solution:

The point $P(2,4,4)$ corresponds to $t = 2$.

1. $$T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{i + 2j + 2tk}{\sqrt{1^2 + 2^2 + (2t)^2}} = \frac{i + 2j + 2tk}{\sqrt{5 + 4t^2}}$$

The unit tangent vector at P is $T(2) = \frac{1}{\sqrt{21}}(i + 2j + 4k)$.

2. $$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

$$T'(t) = \left(\frac{i + 2j + 2tk}{\sqrt{5 + 4t^2}}\right)' = \frac{-4ti - 8tj + 10k}{(5 + 4t^2)\sqrt{5 + 4t^2}}$$

So $$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{-2ti - 4tj + 5k}{\sqrt{20t^2 + 25}}$$

The principal normal vector at P is $N(2) = \frac{1}{\sqrt{105}}(-4i - 8j + 5k)$.

3. Normal vector for the osculating plane:

$$\vec{a} = T(2) \times N(2) = \frac{1}{\sqrt{21}}(i + 2j + 4k) \times \frac{1}{\sqrt{105}}(-4i - 8j + 5k)$$

$$= \frac{1}{21\sqrt{5}}\big(-8(i \times j) + 5(i \times k) - 8(j \times i) + 10(j \times k) - 16(k \times i) - 32(k \times j)\big)$$

$$= \frac{1}{\sqrt{5}}(2i - j)$$

Osculating plane at P: $2(x-2) - (y-4) = 0 \iff 2x - y = 0$.