

NameSectionQuiz 4

① Given the function $f(x,y) = \frac{x^4 - x^2y}{x^4 + y^2}$. Show that f does not have a limit at $(0,0)$.

Hint: Let (x,y) tend to $(0,0)$ along the x -axis and y -axis.

② Find the ~~directional~~ ^{partial} derivatives of the function $f(x,y,z) = \ln(x^2 + y^2 + z^2)$ at $P(1,0,0)$ ~~in the direction~~ $\vec{a} = (1, 2, 2)$.

Solⁿ

① along x -axis: $y=0$ and $x \rightarrow 0$ so $f(x,y) = \frac{x^4}{x^4} = 1$
so the limit along x -axis = 1

along y -axis: $x=0$ and $y \rightarrow 0$ so $f(x,y) = \frac{0}{y^2} = 0$
so the limit along y -axis = 0

since $1 \neq 0$, f does not have a limit at $(0,0)$.

② $\nabla f(x,y,z) = \left(\frac{2x}{x^2 + y^2 + z^2}, \frac{2y}{x^2 + y^2 + z^2}, \frac{2z}{x^2 + y^2 + z^2} \right)$

so $\nabla f(1,0,0) = (2,0,0)$.