09/30/2009

Name :

Section :

Quiz #4.

1. Write an equation in x, y, z for the tangent plane and scalar parametric equations for the normal line at the given point P:

$$z = x^2 + xy + y^2 - 6x + 2 \quad ; \quad P(4,-2,-10)$$

2. Find the critical points of the function

$$f(x,y) = x^3 - 2xy + y^2 \quad ; \quad 0 \le x \le 1 \, , \, 0 \le y \le 1 .$$

Solution :

1. $f(x,y,z) = x^2 + xy + y^2 - 6x - z + 2$

A normal vector to the tangent plane is $\vec{a} = \nabla f(P)$.

$$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$= (2x+y-6)\vec{i} + (x+2y)\vec{j} - \vec{k}$$

$$\nabla f(P) = (2.4 - 2 - 6)\vec{i} + (4 + 2.-2)\vec{j} - \vec{k}$$

$$= -\vec{k}$$

Tangent plane: $\vec{a}.(X-P) = 0$ or

$-\vec{k}.((x,y,z) - (4,-2,-10)) = 0$ or

$z - 10 = 0$

Normal line: $l(t) = (4,-2,-10) + t(-\vec{k})$

2. $\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} = (3x^2 - 2y)\vec{i} + (2y - 2x)\vec{j}$

$$\nabla f = 0 \Leftrightarrow \begin{cases} 2y = 3x^2 \\ 2y = 2x \end{cases} \Leftrightarrow (x,y) = (0,0) \text{ or } \left(\tfrac{2}{3}, \tfrac{2}{3}\right)$$

But $(0,0)$ lies on the boundary so it's not an interior point and hence not a critical point.

The only critical point is $\left(\frac{2}{3}, \frac{2}{3}\right)$.