

Diagonalize  $A = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$ . (i.e. ~~write A as A =~~  
find  $U$  and  $D$  s.t.  $U^t A U = D$ )

Sol<sup>n</sup>: The char poly of  $A$  is  $(3-t)(6-t) - 4 = t^2 - 9t + 14$   
has 2 roots  $\mu_+ = 7$  and  $\mu_- = 2 \Rightarrow D = \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$

We have  $A - \mu_+ I = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}$  so  $r_1 = (-4, 2)$  (or  $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ )

$u_1 = \frac{r_1^\perp}{|r_1|} = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$  and  $u_2 = u_1^\perp = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Hence  $U = [u_1 \ u_2] = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix}$ .