Name:

Section:

Quiz #7

Find the singular value decomposition of

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(You need only to find U and D such that $A = VDU^T$)

Solution:

$$A^TA = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

- Eigenvalues of A^TA:

$$(2-\lambda)^2 - 1^2 = 0$$

$$\lambda_1 = 3; \quad \lambda_2 = 1$$

$$\Rightarrow \sigma_1^2 = 3 \qquad \sigma_2^2 = 1 \Rightarrow \sigma_1 = \sqrt{3}; \ \sigma_2 = 1$$

So $D = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix}$

- Eigenvectors of A^TA:

$$B_1 = A^TA - \sigma_1^2 I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$u_1 = \frac{r_1^\perp}{\|r_1\|} = \frac{1}{\sqrt{2}}(1,1)$$

$$u_2 = u_1^\perp = \frac{1}{\sqrt{2}}(1,-1)$$

So $U = [u_1, u_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$.