Name:

Section:

Quiz 8

1. Evaluate the double integral:

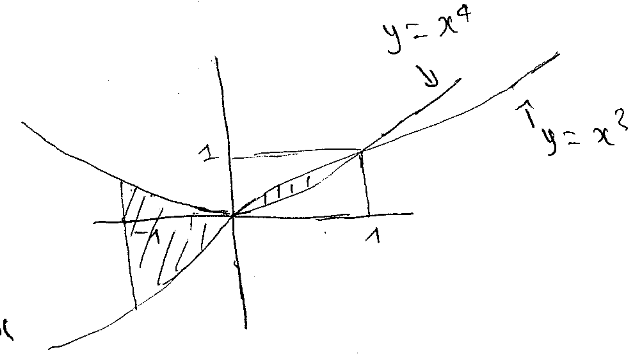
$$\iint_{\Omega} (x+y)\,dx\,dy$$ where Ω is the region between $y = x^3$ and $y = x^4$, $x \in [-1, 1]$

2. Calculate using polar coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} dy\,dx$$

Solution:

1:



$$\iint_{\Omega} (x+d)\,dx\,dy =$$

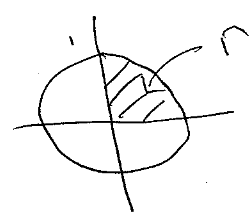
$$\int_{-1}^{0}\int_{x^3}^{x^4} (x+d)\,dy\,dx + \int_0^1 \int_{x^4}^{x^3} (x+y)\,dy\,dx$$

$$= \int_{-1}^{0} \left[xy + \frac{y^2}{2} \right]\Big|_{x^3}^{x^4} dx + \int_0^1 \left[xy + \frac{y^2}{2} \right]\Big|_{x^4}^{x^3} dx$$

$$= \int_{-1}^{0} \left(x^5 + \frac{x^8}{2} - x^4 - \frac{x^6}{2}\right) dx + \int_0^1 \left(x^4 + \frac{x^6}{2} - x^5 - \frac{x^8}{2}\right) dx$$

$$= \left[\frac{x^6}{6} + \frac{x^9}{18} - \frac{x^5}{5} - \frac{x^7}{14}\right]\Big|_{-1}^{0} + \left[\frac{x^5}{5} + \frac{x^7}{14} - \frac{x^6}{6} - \frac{x^9}{18}\right]_0^1 = -\frac{1}{3}.$$

2. $\begin{matrix} 0 \le x \le 1 \\ 0 \le y \le \sqrt{1-x^2} \end{matrix} \iff \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \\ x^2+y^2 \le 1 \end{matrix} \Rightarrow r =$



$\begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix} \Rightarrow \begin{matrix} 0 \le r \le 1 \\ 0 \le \theta \le \frac{\pi}{2} \end{matrix}$

$$\int_0^1\int_0^{\sqrt{1-x^2}} dy\,dx = \iint_{r} r\,dr\,d\theta = \int_0^{\pi/2}\left(\int_0^1 r\,dr\right)d\theta = \int_0^{\pi/2} \frac{1}{2}\,d\theta = \frac{\pi}{4}.$$