Fall '09 Quiz 9. Math 2605D

Find the volume of the first-octant solid bounded by the planes $z = x$, $y - x = 2$ and the cylinder $y = x^2$.

Sol$^{\underline{n}}$.

T is $\begin{cases} 0 \leq z \leq x \\ (x,y) \in \Omega_{xy} \text{ defined by} \end{cases}$ • $y = x + 2$ and $y = x^2$.
• $x \geq 0$, $y \geq 0$.

Points of intersection $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$
$\Rightarrow x = 2$ or $x = -1$

Since $x \geq 0$, we take $x = 2$ only and therefore $0 \leq x \leq 2$

When $x = 1 \in [0,2]$: $\begin{cases} x^2 = 1 \\ x + 2 = 3 \end{cases} \Rightarrow x + 2 > x^2$

so $x^2 \leq y \leq x + 2$

Hence T is • $0 \leq z \leq x$
• $\left.\begin{array}{l} 0 \leq x \leq 2 \\ x^2 \leq y \leq x + 2 \end{array}\right\} \Omega_{xy}$

and

$$\mathrm{vol}(T) = \int_0^2 \left(\int_{x^2}^{x+2} \left(\int_0^x dz \right) dy \right) dx$$

$$= \int_0^2 \left(\int_{x^2}^{x+2} x \, dy \right) dx$$

$$= \int_0^2 x (x + 2 - x^2) \, dx$$

$$= \frac{x^3}{3} + x^2 - \frac{x^4}{4} \Big|_0^2$$

$$= \frac{8}{3} + 4 - 4 = \frac{8}{3}.$$