Homework 6

Page 61 of Rosenlicht: 18, 19, 23.

Additional Problems:

1. Show that $\lim s_n = +\infty$ if and only if $\lim(-s_n) = -\infty$.

2. Suppose that there exists a N_0 such that if $n \geq N_0$ then $s_n \leq t_n$.

   (a) Prove that if $\lim s_n = +\infty$ then $\lim t_n = +\infty$;

   (b) If $\lim s_n = s$ and $\lim t_n = t$, then $s \leq t$.

3. Show that $\liminf s_n = -\limsup(-s_n)$.

4. Let $\{s_n\}$ be a sequence of real numbers and let

$$\sigma_n = \frac{1}{n}\sum_{k=1}^{n} s_k.$$

   (a) Show that $\liminf s_n \leq \liminf \sigma_n \leq \limsup \sigma_n \leq \limsup s_n$;

   (b) Show that if $\lim s_n$ exists, then $\lim \sigma_n$ exists and $\lim s_n = \lim \sigma_n$.

5. Prove that a sequence $\{s_n\}$ of real numbers is bounded if and only if $\limsup |s_n| < +\infty$.