Homework 5

Due Date: March 12, 2013.

Page 160 of Rosenlicht: 1, 2, 3, 4, 20, 35.

Additional Problems:

1. Let f_k be a sequence of bounded Riemann integrable functions defined on $[a, b]$. Suppose that $f_k \to f$ uniformly. Then prove that f is integrable on $[a, b]$ and

$$\int_a^b f_k(x)dx \to \int_a^b f(x)dx.$$

2. Prove that if $\sum_k g_k$ is a series of functions that converges uniformly on a set S and if h is a bounded function on S, then

$$\sum_k hg_k$$

converges uniformly on S.

3. Let $s(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$ and $c(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots$ for all $x \in \mathbb{R}$.

(a) Prove that $s' = c$ and $c' = -s$.

(b) Prove that $(s^2 + c^2)' = 0$.

(c) Prove that $s^2 + c^2 = 1$.