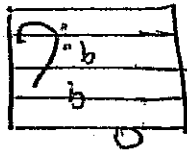


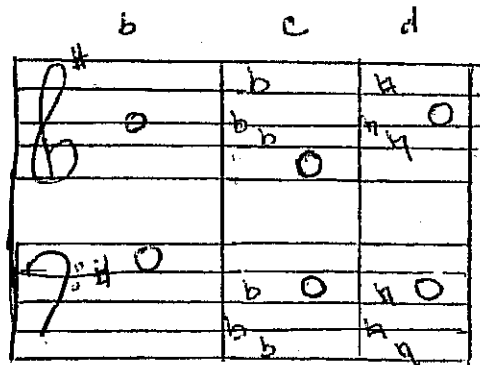
1 a $12 \log_2(3) \approx 19$ semitones



b $12 \log_2(\frac{2}{3}) \approx -16$ semitones

c $12 \log_2(2.3) \approx 14$ semitones

d $12 \log_2(\pi^{-1}) \approx -20$



2 a $19 \log_2 \frac{5}{2} \approx 25.12$

b $8 \log_2 3 \approx 12.68$

c $13 \log_2 .85 \approx -3.05$

d $4 \log_2 2\pi \approx 10.61$

- 3
- a $11 \log_2 \frac{3}{2} \approx 6.43 \approx 6$ units $.43$ units $(\frac{1200 \text{ cents}}{11 \text{ units}}) \approx 47$ cents error
 - b $17 \log_2 \frac{3}{2} \approx 9.94 \approx 10$ units $.06$ units $(\frac{1200 \text{ cents}}{17 \text{ units}}) \approx 4$ cents error
 - c $21 \log_2 \frac{3}{2} \approx 12.28 \approx 12$ units $.28$ units $(\frac{1200 \text{ cents}}{21 \text{ units}}) \approx 16$ cents
 - d $12 \log_2 \frac{3}{2} \approx 7.02 \approx 7$ 2 cents error
- 17 is the best of the three, but not better than 12

4 a (\mathbb{Z}, \cdot) is a monoid. \mathbb{Z} is closed under \cdot , \cdot is associative and $1 \in \mathbb{Z}$ is the identity. It is not a group because $2 \in \mathbb{Z}$ but $2^{-1} = \frac{1}{2} \notin \mathbb{Z}$.

b $(\mathbb{Z}, +)$ is a group. \mathbb{Z} is closed under $+$, $+$ is associative $0 \in \mathbb{Z}$ is the identity and for $x \in \mathbb{Z}$, $-x \in \mathbb{Z}$ is the inverse.

c $(\mathbb{Z}_1, -1, \cdot)$ is a group. \cdot is associative, From the table we can see that $\mathbb{Z}_1, -1$ is closed under \cdot , $1 \in \mathbb{Z}_1, -1$ is the identity and $1^{-1} = 1 \in \mathbb{Z}_1, -1$ and $-1^{-1} = -1 \in \mathbb{Z}_1, -1$

	1	-1
1	1	-1
-1	-1	1

d $(\mathbb{Z}^{-1}, 0, 1, +)$ is not a monoid. $1+1=2 \notin \mathbb{Z}^{-1}, 0, 1$

5

$$f(x+y) = b^{x+y} = b^x b^y = f(x) \cdot f(y)$$

$$g(x \cdot y) = \log_b(x \cdot y) = \log_b(x) + \log_b(y) = g(x) + g(y)$$

$$f(g(x)) = f(\log_b(x)) = b^{\log_b(x)} = x$$

$$g(f(x)) = g(b^x) = \log_b b^x = x \log_b b = x$$

When this is applied to $f(x) = 2^{x/12}$ this shows that measurement of intervals in semitones is the same as (isomorphic to) measurement as a frequency ratio.

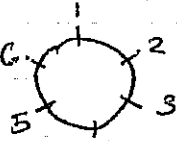
6 a $[14] + [23] - [37] = [1]$

b $[1] + [7] + [4] = [18] = [6]$

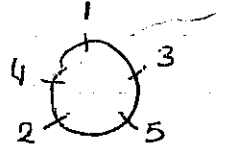
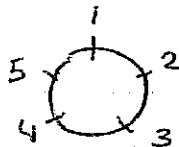
c $6 \cdot [7] = [42] = [6]$

d $3 \cdot [3] - 6 \cdot [2] = [-3] = [9]$

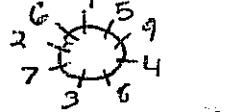
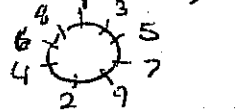
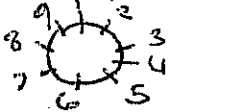
7 a $n=6 \quad \phi(6) = \{1, -1=5\}$



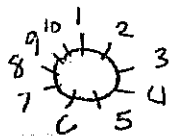
b $n=5 \quad \phi(5) = \{1, -1=4, 2, -2=3\}$



c $n=9 \quad \phi(9) = \{1, -1=8, 2, -2=7, 4, -4=5\}$



d $n=10 \quad \phi(10) = \{1, -1=9, 3, -3=7\}$



8

A	F	C#	D	A#	C	B	E	D#	G	F#	G#
C#	A	F	F#	D	E	B#	G#	G	B	A#	C
F	C#	A	A#	F#	G#	G	C	B	D#	D	F#
E	C	G#	A	F	G	F#	B	A#	D	C#	D#
G#	E	C	C#	A	B	A#	D#	D	F#	F	G
F#	D	A#	B	G	A	G#	C#	C	E	D#	F#
G	D#	B	C	G#	A#	A	D	C#	F	E	F#
D	A#	F#	G	D#	F	E	A	G#	C	B	C#
D#	B	G	G#	E	F#	F	A#	A	C#	C	D
B	G	D#	E	C	D	C#	F#	F	A	G#	A#
C	G#	E	F	C#	D#	D	G	F#	A#	A	B
A#	F#	D	D#	B	C#	C	F	E	G#	G	A



9a

0	2	1	b	0	4	2	3	1
1	0	2		1	0	3	4	2
2	1	0		3	2	0	1	4
				2	1	4	0	3
				4	3	1	2	0

c

0	2	4	1	3	5
4	0	2	5	1	3
2	4	0	3	5	1
5	1	3	0	2	4
3	5	1	4	0	2
1	3	5	2	4	0

d

0	5	6	3	2	1	4
2	0	1	5	4	3	6
1	6	0	4	3	2	5
4	2	3	0	6	5	1
5	3	4	1	0	6	2
6	4	5	2	1	0	3
3	1	2	6	5	4	0

MOONLIGHT SONATA

(First Movement)

Dedicated to Countess Julia Guicciardi

By LUDWIG VAN BEETHOVEN

Adagio sostenuto (♩ = 52)
sempre *pp* e con sordini

Handwritten annotations: *una corda* in the bass staff; *1* and *2* in the bass staff; *C#m* and *C#m7* in the bass staff.

Handwritten annotations: *3* in the bass staff; *A*, *Dm*, *C#m*, *G#7*, and *C#m* in the bass staff; *pp* in the bass staff.

Handwritten annotations: *6* and *7* in the bass staff.

Handwritten annotations: *5 4 3* in the treble staff; *1*, *2*, *1 2* in the bass staff; *una corda* in the bass staff; *8*, *9*, and *10* in the bass staff.

Handwritten annotations: *11* in the bass staff; *4* and *5* in the bass staff.