EXAM I

Math 109 / Music 109A. Spring 2006

Name ________________________________ Id __________________________

Each problem is worth 10 points.

(1) Aural: Notate the rhythm (one measure each).

(a) \[\frac{4}{4}\] (b) \[\frac{12}{8}\]

Circle the interval between the two notes.

(c) minor third  \quad (d) minor third
   major third  \quad fifth
   fifth        \quad octave
   octave

Circle the triad type.

(e) minor  \quad (e) major
   major  \quad diminished
   diminished  \quad augmented
   augmented

(2) Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

(a) \[f(x) = -(x - 1)^2\]  \quad (b) \[g(x) = 1 + \sin 2x\]
(3) For the following pairs of integers $m, n$, find the numbers $q$ and $r$ whose existence is asserted in the division algorithm ($n = qm + r$):

(a) $3, 77$

(b) $11, -55k + 16$, where $k$ some integer.

(4) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

(a) \[ \boxed{\text{B}_4^4} \] \hspace{1cm} (b) \[ \boxed{\text{G}_2} \] \hspace{1cm} (c) \[ \boxed{\text{A}_5^4} \]

(5) For the set $\{(a, b) \in \mathbb{Z}^2 | b \neq 0\}$ show that the relation $\sim$ defined by $(a, b) \sim (a', b')$ iff $ab' - a'b = 0$ is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers $\mathbb{Q}$.

OR

For the set $\mathbb{Z}$ and a fixed positive integer $m$, show that the relation $\equiv$ defined by $k \equiv \ell$ iff $m \mid k - \ell$ is an equivalence relation. Explain why there are exactly $m$ equivalence classes.
(6) For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Locrian with tonic F♯

(c) Lydian with tonic E♭

(7) Add the needed sharps or flats to notes so that the following gives the Phrygian scale tones 1 to 8, from C to C.

(8) Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic down one scale tone in the second measure

(b) chromatic up a minor third (from the original) in the third measure

(9) Give the duration in beats of:

(a) a doubly-dotted eighth note in 2/4 time.

(b) a quarter note in 6/8 time (compound time signature).

(c) an eighth note 7-tuplet in 4/4 time.
(10) For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

Mary had a little lamb, little lamb,

little lamb, Mary had a little lamb, his

fleece was white as snow