(1) **Aural:** Notate the rhythm (one measure each).

(a) \[ \begin{aligned} \frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \end{aligned} \] (b) \[ \begin{aligned} \frac{12}{12} \quad \frac{12}{12} \quad \frac{12}{12} \quad \frac{12}{12} \quad \frac{12}{12} \end{aligned} \]

Circle the interval between the two notes.

(c) minor third
(d) minor third

Circle the triad type.

(e) minor

(2) Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

(a) \[ f(x) = -(x - 1)^2 \]

(b) \[ g(x) = 1 + \sin 2x \]
(3) For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( (n = qm + r) \):

(a) 3, 77; \[ 77 = 25 \cdot 3 + 2 \] \( q = 25 \), \( r = 2 \)

(b) 11, \(-55k + 16\), where \( k \) some integer.
\[ -55k + 16 = (-5k+1) \cdot 11 + 5 \] \( q = -5k + 1 \), \( r = 5 \)

(4) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

(a) \[ B_4^b \] (b) \[ A_5^b \] (c)

(5) For the set \( \{ (a, b) \in \mathbb{Z}^2 | b \neq 0 \} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) iff \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

**OR**

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) if \( m | k - \ell \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.

1) refl. \( a - ab = 0 \), so \( (a, b) \sim (a, b) \)
2) symm. if \( (a, b) \sim (a', b') \) then \( ab' = a'b \)
3) trans. if \( (a, b) \sim (a', b') \) and \( (a', b') \sim (a'', b'') \) then
   \[ a' b'' = a'' b' \]
   \[ a'' b'' = a'' b' \]
   \[ a' b'' = a'' b' \]
   So \( ab'' = a'' b' \). Divide \( b' (\neq 0) \) to get \( a'b = a'b' \)

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The association \( (a, b) \rightarrow \frac{a}{b} \) gives a \( 1\)-\( 1 \) relationship of equivalence classes \( \leftrightarrow \mathbb{Q} \).

1) left \( k - k = 0 \), so \( k \equiv k \)
2) symm. \( k \equiv \ell \), then \( k - \ell = hm \), so \( k - k = (-h)m \)
(6) For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Locrian with tonic F♯

(c) Lydian with tonic Eb

(7) Add the needed sharps or flats to notes so that the following gives the Phrygian scale tones 1 to 8, from C to C.

(8) Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic down one scale tone in the second measure
(b) chromatic up a minor third (from the original) in the third measure

(9) Give the duration in beats of:

(a) a doubly-dotted eighth note in \( \frac{3}{4} \) time.

(b) a quarter note in \( \frac{6}{8} \) time (compound time signature).

(c) an eighth note 7-tuplet in \( \frac{4}{4} \) time.
(10) For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

A B A C

ms 2, 3, 4  

lyric translation

ms 2, 3  

melodic transposition