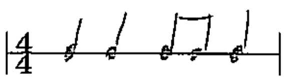
EXAM I

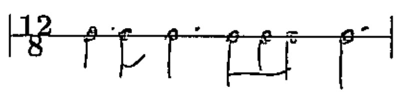
Math 109 / Music 109A, Spring 2009

Name Key Id ______________

Each problem is worth 10 points.

1. Aural: Notate the rhythm (one measure each).

   (a) 

   (b) 

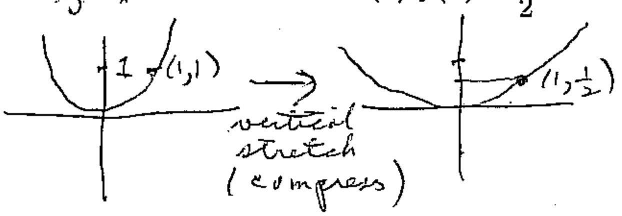
   Circle the triad type.

   (c) major D
   minor
   diminished

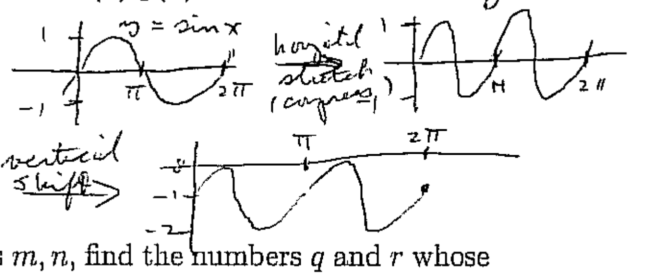
   (d) minor
   minor
   diminished F dim

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   $y = x^2$ (a) $f(x) = \frac{1}{2}x^2$

   (b) $g(x) = -1 + \sin 2x$ $y = \sin 2x$

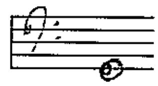
   

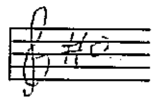
3. For the following pairs of integers m, n, find the numbers q and r whose existence is asserted in the division algorithm ($n = qm + r$):

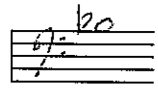
   (a) $11, -101$; $-101 = -10 \cdot 11 + 9$ $q = -10$, $r = 9$

   (b) $5, 3035\ell + 9$, where ℓ some integer. $3035\ell + 9 = (607\ell + 1)5 + 4$
   $q = 607\ell + 1$, $r = 4$

4. Write the indicated note as a whole note on the given staff, <u>choosing</u> <u>and</u> <u>notating</u> <u>an</u> <u>appropriate</u> <u>clef</u>.

(a)  G_2

(b)  $C_5^\sharp$

(c)  $B_3^\flat$

5. For the set $\{(a,b) \in \mathbb{Z}^2 \,|\, b \neq 0\}$ show that the relation $\sim$ defined by $(a,b) \sim (a',b')$ if and only if $ab' - a'b = 0$ is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers $\mathbb{Q}$.

<u>OR</u>

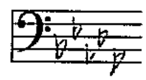
For the set $\mathbb{Z}$ and a fixed positive integer m, show that the relation $\equiv$ defined by $k \equiv \ell$ if and only if $m \,|\, (k - \ell)$ is an equivalence relation. Explain why there are exactly m equivalence classes.

First: $(a,b) \sim (a,b)$, since $ab - ab = 0$, so $\sim$ is reflexive. If $(a,b) \sim (a',b')$ then $ab' - a'b = 0$, so $a'b - ab' = 0$ (multiply by -1), so $(a',b') \sim (a,b)$. Thus $\sim$ is symmetric. If $(a,b) \sim (a',b')$ and $(a',b') \sim (a'',b'')$, then (1) $ab' - a'b = 0$ and (2) $a'b'' - a''b' = 0$. Want to show $ab'' - a''b = 0$.
Multiply (1) by b'', (2) by b to get
$$(ab' - a'b)b'' + (a'b'' - a''b')b = 0$$
$$ab'b'' \underbrace{- a'bb'' + a'b''b}_{\text{cancel}} - a''b'b = 0$$
Dividing by b' $(\neq 0)$ we get $ab'' - a''b = 0$ as desired, showing that $(a,b) \sim (a',b')$. So $\sim$ is transitive.
Define $\varphi : S/\sim \;\to \mathbb{Q}$ by $(a,b) \mapsto \frac{a}{b}$. This is well-defined, since if $(a,b) \sim (a',b')$ we have $\frac{a}{b} = \frac{a'}{b'}$, which is exactly the same as saying $ab' - a'b = 0$. Thus φ is ~~injective~~ one to one. Clearly every $\frac{m}{n} \in \mathbb{Q}$ is in the image, so φ is onto as well.

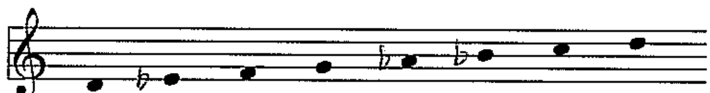
Second: $m \,|\, (k-k) = 0$, so $\equiv$ is reflexive. If $k \equiv \ell$ and $\ell \equiv h$, then $k - \ell = am$, $\ell - h = bm$. So $k - \ell + \ell - h = am + bm$ i.e. $k - h = (a+b)m$. So $m \,|\, (k-h)$ so $k \equiv h$, so $\equiv$ is transitive. If $k \equiv \ell$, then $am = k - \ell$, so $-am = \ell - k$ showing $\ell \equiv k$. So $\equiv$ is symmetric.
The classes $\{0\}, \ldots, \{m-1\}$ are distinct, since any two of $0, \ldots, m-1$ differ by less than m. For any $n \in \mathbb{Z}$, write $n = qm + r$ using Division algorithm. Then $n \equiv r$ and $r \in \{0, \ldots, m-1\}$.

6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D 

(c) Aeolian with tonic $B^{\flat}$ 

7. Add the needed sharps or flats <u>to notes</u> so that the following gives the <u>Locrian</u> scale tones $\hat{1}$ to $\hat{8}$, from D to D. (Do <u>not</u> alter D.)



$\hat{1}$ $\hat{2}$ $\hat{3}$ $\hat{4}$ $\hat{5}$ $\hat{6}$ $\hat{7}$ $\hat{8}$

8. Extend the following melody with two measures having the <u>same rhythm</u>, employing the following transformations. Do not write in a key change.

(a) diatonic up one scale tone in the second measure

(b) chromatic up a major third (from the original) in the third measure

9. Give the total duration in beats of:

(a) a doubly-dotted half note in $\frac{4}{4}$ time. $= 2 \qquad = 2(1 + \frac{1}{2} + \frac{1}{4}) = 2 \cdot \frac{7}{4} = \frac{7}{2}$

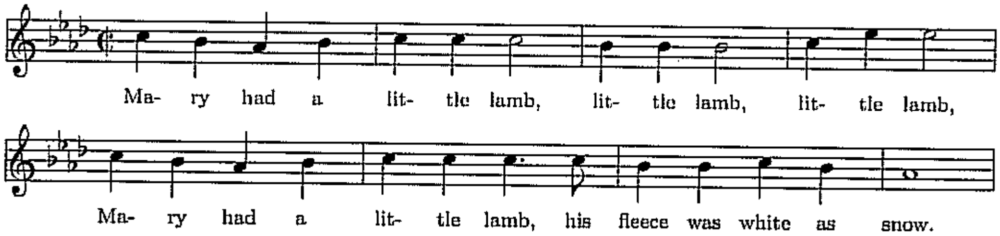
(b) a half note in $\frac{9}{8}$ time (compound time signature). $= 1 \qquad = \frac{4}{3}$

(c) a sixteenth note 9-tuplet in $\frac{4}{4}$ time.

$= \frac{1}{2^4} \quad 4 = n + r \quad 2^3 = 8 < 9 < 16$, so $r = 3$.

So $n = 1$. Duration is that of $\frac{1}{2^1}$-note $=$, which is 2 beats.

10. For the song *Mary Had A Little Lamb,* give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.



A B A C

rhythmic translation: m 2 → 3, → 4

diatonic transposition: m 2 → 3