

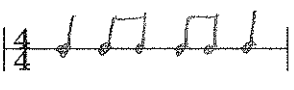
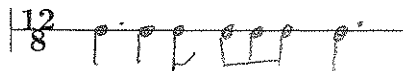
EXAM I

Math 109 / Music 109A, Spring 2011

Name Solutions Id _____

Each problem is worth 10 points.

1. Aural: Notate the rhythm (one measure each).

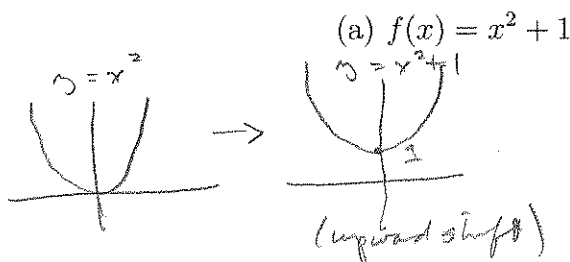
(a)  (b) 

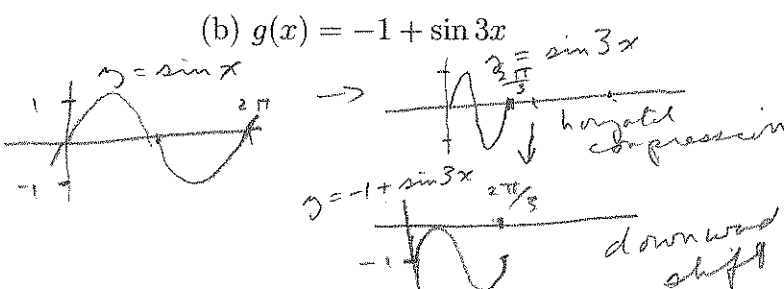
Circle the triad type.

(c) major
minor (A^b_m)

(d) major (F)
minor

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

(a) $f(x) = x^2 + 1$ 

(b) $g(x) = -1 + \sin 3x$ 

3. For the following pairs of integers m, n , find the numbers q and r whose existence is asserted in the division algorithm ($n = qm + r$):

(a) $11, -23$ $-23 = -3 \cdot 11 + 10$ $q = -3, r = 10$

(b) $3, 42d + 5$ (where d is some integer)

$42d + 5 = (14d + 1) \cdot 3 + 2$ $q = 14d + 1, r = 2$

4. Write the indicated note as a whole note, choosing and notating an appropriate clef.

(a)  B_2

(b)  G_5^\sharp

(c)  E_4^b

5. For the set \mathbb{Z} and a fixed positive integer m , show that the relation \equiv defined by $k \equiv l$ if and only if $m \mid (k - l)$ is an equivalence relation. Explain why there are exactly m equivalence classes.

reflexive: $k - k = 0 = 0 \cdot m$, so $m \mid (k - k)$ so $k \equiv k$.

symmetric: If $k \equiv l$, then $m \mid (k - l)$ so $k - l = a \cdot m$. Then $l - k = (-a)m$, so $m \mid (l - k)$. Hence $l \equiv k$.

transitive: If $k \equiv l$ and $l \equiv t$, then $m \mid (k - l)$ and $m \mid (l - t)$. So $k - l = a \cdot m$ and $l - t = b \cdot m$. Adding these equations gives $k - t = (a + b) \cdot m$, so $m \mid (k - t)$. Hence $k \equiv t$.

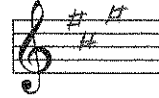
Classes: For any $n \in \mathbb{Z}$ we can write $n = qm + r$ with $0 \leq r < m$. Then $m \mid (n - r)$ so $n \equiv r$.

This shows that $[n] \stackrel{m}{=} [r]$ is one of the classes $\{ [0], [1], \dots, [m-1] \}$.

These classes are distinct, for if $0 \leq r < r' < m$, then $r' - r$ cannot be divisible by m , so $r \neq r'$. Hence the list $\{ [0], [1], \dots, [m-1] \}$ is complete and its entries are distinct. There are m classes.

6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D



(c) Phrygian with tonic C#



7. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., E^{b7}). Below the staff, label each chord by root scale tone (e.g. bIII⁷).

Handwritten labels above the staff: Bm, F#7, DM7, C#7, F#

Handwritten labels below the staff: Im, V⁷, III^{b7}, II^{b7}, V

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure



9. Give the total duration in beats of:

- (a) a doubly-dotted quarter note in $\frac{2}{2}$ time. $\downarrow d = \frac{1}{2}$ beat
 $\frac{1}{2}(1 + \frac{1}{2} + \frac{1}{4}) = \frac{1}{2} \cdot \frac{7}{4} = \boxed{\frac{7}{8} \text{ beat}}$
- (b) a half note in $\frac{9}{8}$ time (compound time signature). $\downarrow d = 1$ beat, $\downarrow h = \frac{1}{3}$ beat
 $d = d + d + d = 1 + \frac{1}{3} = \boxed{\frac{4}{3} \text{ beat}}$
- (c) an eighth note quintuplet in $\frac{4}{4}$ time.
 $\frac{1}{8} = \frac{1}{2^3}$ $n+r=3$ $k=5$ $2^2 < 5 < 2^3$ so $r=2$
 $\frac{1}{21}$ - note = d half note, $\boxed{2 \text{ beats}}$

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

Ma- ry had a lit- tle lamb, lit- tle lamb, lit- tle lamb,
 Ma- ry had a lit- tle lamb, his fleece was white as snow.

A B A C
 rhythmic translation m5, 2, 3, 4
 melodic translation m5, 2, 3