EXAM II

Math 109 / Music 109A, Spring 2009

Name ______________________________ Id ____________________

Each problem is worth 10 points.

1. Express each of these musical intervals as an element of $\mathbb{R}^+$ three ways: (1) as a power of 2, (2) as a radical or the reciprical of a radical, and (3) by a decimal approximation.

   (a) up 65 cents

   (b) down a keyboard tritone

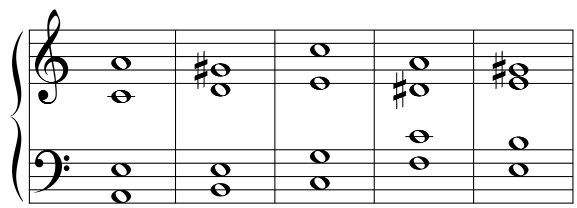
2. Convert to the specified additive measurement the intervals given by the following ratios. Round off to 2 digits to the right of the decimal and indicate whether the interval is upward or downward.

   (a) 6/5, convert to semitones

   (b) π, convert to cents

3. A string on a stringed instrument has length 100 cm. Indicate the positions of the single fret which will allow the string to play the note (a) a keyboard major third above the original pitch, and (b) a ratio 5/4 above the original pitch.

4. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., $E^{\flat 7}$). Below the staff, label each chord by root scale tone (e.g. $\flat III^7$). Also, one of the chords could be considered misspelled. Which chord is it?



5. Evaluate these logarithms <u>without a calculator</u>. Write down each step of the simplification. You may express your answer as a fraction.

   (a) $\log_3 \left(\frac{81}{\sqrt{3}} \right)$

   (b) $\log_b \left(\frac{b^p}{\sqrt[m]{b^\ell}} \right)$

6. Write on the staff the note which best approximates the frequency having the given interval ratio $r = 7/24$ from the given note. Compute the error in cents.

7. Determine $\phi(9)$ (ϕ is the Euler phi function) by listing all the generating intervals in the 9-chromatic scale. Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

8. Determine whether or not the pair $(\mathbb{R}\,,\,\cdot)$ is a monoid. If so, is it also also a group? Justify your answer.

9. Let $b > 1$ be a real number. Explain why the functions $f(x) = b^x$ and $g(x) = \log_b(x)$ are group homomorphisms, and why they are inverse to each other, thereby giving isomorphisms between the groups $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$. Explain the connection of this with musical intervals.

10. Create a 5-tone row chart for the original row $([0], [4], [1], [3], [2])$ in $\mathbb{Z}_5$