

EXAM II

Math 109 / Music 109A, Spring 2011

Name _____ Id _____

Each problem is worth 10 points. Round off each decimal approximation to two digits to the right of the decimal.

- Express each of these musical intervals as an element of \mathbb{R}^+ three ways:
(1) as a power of 2, (2) as a radical or the reciprocal of a radical, and
(3) by a decimal approximation.
 - up 56 cents
 - down a keyboard fourth
- Convert to the specified additive measurement the intervals given by the following ratios.
 - $7/4$, convert to semitones
 - $\pi^2/5$, convert to cents
- A string on a stringed instrument has length 100 cm. Indicate the positions of the single fret which will allow the string to play the note
(a) a keyboard major third above the original pitch, and (b) a ratio $5/4$ above the original pitch.

4. Complete the following to a four-part harmonization of the given melody, major mode, using only whole notes, so that the melody is the top part, the lowest note is always the root, and the result has two parts on each staff. The chords should be the those indicated under the staff.

5. Evaluate these logarithms without a calculator. Write down each step of the simplification. You may express your answer as a fraction.

(a) $\log_7 \left(\frac{\sqrt[3]{49}}{7} \right)$

(b) $\log_b \left(\frac{1}{\sqrt[n]{b^k}} \right)$

6. Write on the staff the note which best approximates the frequency having the given interval ratio $r = 3/25$ from the given note. Compute the error in cents.

7. Determine $\phi(8)$ (ϕ is the Euler phi function) by listing all the generating intervals in the 8-chromatic scale. Indicate which pairs of generating intervals are inverse to each other and for each pair draw the circle of intervals which is based on one element of the pair in the clockwise direction, the other element of the pair in the counterclockwise direction.

8. Determine whether or not each of the following pairs forms a monoid. If so, is it also a group? Justify your answers.

(a) $(\mathbb{Q} - \{0\}, \cdot)$

(b) $(\mathbb{Z}, +)$

(c) (\mathbb{Z}_4, \cdot)

9. Explain why the functions $f(x) = 2^{x/12}$ and $g(x) = 12 \log_2(x)$ are group homomorphisms, and why they are inverse to each other, thereby giving isomorphisms between the groups $(\mathbb{R}, +)$ and (\mathbb{R}^+, \cdot) . Explain the connection of this with musical intervals.

10. Create a 6-tone row chart for the original row $([0], [2], [5], [1], [4], [3])$ in \mathbb{Z}_6