EXAM III
Math 109 / Music 109A, Spring 2006

Name _____________________________  Id __________________

Each problem is worth 10 points.

(1) Prove that there are infinitely many prime numbers.

(2) Find the period, frequency, amplitude, and phase shift for the function
    \[ f(t) = 3 \sin(1000\pi t + \frac{\pi}{4}) \]
    and express it in the form \( A \sin(\alpha t) + B \cos(\alpha t) \).

(3) Find the period, frequency, amplitude, and phase shift for the function
    \[ g(t) = 3 \sin(250\pi t) + 2 \cos(250\pi t) \]
    and express it in the form \( d \sin(\alpha t + \beta) \).
(4) Find the value \( \alpha \) for which the pitch associated to the periodic function \( y = \sin(\alpha t) \), where \( t \) is time in seconds, is \( B_2^5 \).

(5) Suppose a musical tone with pitch \( B_2^5 \) has harmonics 1 and 3 only, with amplitudes 1 and \( \frac{1}{6} \), respectively, and phase shifts 0 and \( \pi/2 \), respectively. Suppose also that the vertical shift \( C \) is 0. Write its Fourier series as two summands each having the form \( A_k \sin(kt) + B_k \cos(kt) \). (Use your answer to (4).)

(6) Is:

(a) a just fourth plus a just minor third equal to a just minor sixth?

(b) two lesser whole tones equal to a just major third?

(c) the difference between a just minor third and a septimal minor third equal to the comma of Didymus?

In each case above, justify your answer by multiplication and division in \( \mathbb{Q}^+ \). If the answer is no, calculate the difference in cents.

(7) In the just intonation scale, give the reduced integer ratio \( n_1 : n_2 : \cdots \) for each of these diatonic chords:

(a) \( V \)

(b) \( VI\text{m}^7 \)

(c) \( VII\text{dim} \)
(8) Give a just tuning, i.e., a reduced ratio, for this jazz chord which involves “exotic” primes, i.e., primes which are $\geq 7$.

\[
\begin{array}{c}
\text{\\hspace{1cm}} \\
\includegraphics[width=0.3\textwidth]{chord.png}
\end{array}
\]

(9) Assume we have tuned the Pythagorean chromatic scale with the comma placed between VII and $\nu V$. Certain of the major triads with perfect fifths have better thirds by virtue of the fact that the third lies across the comma from the root and fifth.

(a) Express this third in terms of its prime factorization as a rational number.

(b) Calculate the difference between this third from the just third in cents.

(c) Identify by Roman numeral which major triads possess this property.

(10) In the key of A major, write the sequence of five chords such that (a) the last chord is the tonic triad, and (b) the other four chords are seventh chords resolving around the circle of fifths. Identify the chords by the note class of the root, not by Roman numeral.