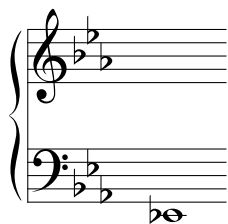




4. Prove that if  $y = f(t)$  has period  $P$ , then  $y = cf(t)$  has period  $P$ , but  $y = f(ct)$  ( $c \neq 0$ ) has period  $P/c$ .

5. On the staff system below, write the keyboard's best approximation for each harmonic up through 11 for the indicated note. For harmonics 5, 7 and 11, indicate how sharp or flat (to the nearest cent) the keyboard's approximation is.



6. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = \sin(880\pi t) + 2 \cos(880\pi t)$$

and express it in the form  $d \sin(\alpha t + \beta)$ , giving a decimal approximation for  $\beta$ .

7. Suppose a musical tone is sounding  $A_2$ , and suppose its second harmonic has amplitude  $1/2$  and phase shift  $\pi/2$ . Express the second harmonic in the form  $g(t) = A \sin \gamma t + B \cos \gamma t$ , for explicit numbers  $A, B$ , and  $\gamma$ .

8. We established that the square wave, defined on  $[0, 2\pi)$  by

$$s(t) = \begin{cases} 1, & \text{for } 0 \leq t < \pi \\ -1, & \text{for } \pi \leq t < 2\pi \end{cases}$$

has Fourier series

$$s(t) = \sin t + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \dots$$

Give the values of the Fourier coefficients  $C, A_k, B_k$  for  $k \in \mathbb{Z}^+$ , indicate the phase shift of each harmonic, and explain why the value of  $B_1$  is such.

9. A certain vowel sound has a formant which amplifies frequencies within 300 Hz of 2500 Hz. A singer sings the vowel at  $C_3$ . Which harmonics are amplified?
10. Express each of these intervals two ways: as rational numbers and in cents, rounding off the latter at 2 digits to the right of the decimal.
- (a) the greater whole tone
  - (b) the just minor third