(1) Add the needed sharps or flats to notes so that the following gives the Dorian scale tones 1 to 8, from F to F.

\[ \text{\textit{Dorian scale}} \]

(2) Complete each excerpt with a measure that repeats the rhythm of the first measure, employing:

(a) **diatonic transposition down** one scale tone.

(b) **chromatic transposition up** a minor third.

(3) Give the (total) duration in beats of:

(a) a dotted eighth note in \( \frac{3}{8} \) time.

\[ \begin{align*}
\text{dotted eighth note} & = \frac{3}{8} \\
\text{rhythm} & = \frac{3}{8} \left( \frac{1}{4} + \frac{1}{4} \right) = \frac{3}{8} \cdot \frac{3}{2} = \frac{9}{16} \\
\end{align*} \]

(b) a sixteenth note 9-tuplet in \( \frac{3}{4} \) time.

\[ \begin{align*}
\text{sixteenth note} & = \frac{1}{16} \\
\text{9-tuplet} & = \frac{9}{16} \\
\text{total} & = \frac{9}{16} + \frac{1}{16} = \frac{10}{16} = \frac{5}{8} \\
\end{align*} \]
(4) Given that $A_4$ is tuned to 440 Hz, give the frequencies of these pitches:

(a) the pitch which is 35 cents sharp of $F_4$

$$2 \cdot 2^{\frac{35}{1200}} = 2 \cdot 1.006944 = 2.013888 \approx 2.01$$

(b) the pitch which is a comma of Pythagoras above middle C

$$2 \cdot 2^{\frac{3}{440}} \cdot \left(2^{\frac{3}{2}} \right) \approx 2.6519$$

Use properties of logarithms to express in terms of logarithms of prime numbers:

$$\log_b \left( \left( \frac{\sqrt{13}}{18} \right)^{\frac{3}{2}} \right) = \frac{3}{2} \log_b 13 - \frac{3}{2} \log_b 18 = \frac{3}{2} \log_b 3 + \frac{3}{2} \log_b 2$$

Express each of these intervals two ways: as real number ratios using integers or quotients of integers with possible radicands, and in cents, rounding off the latter at 2 digits to the right of the decimal.

(a) the septimal minor seventh $\frac{7}{4} = 1.75 \quad 200.83$ cents

(b) the keyboard whole step $\sqrt{2} \approx 1.22 \quad 200$ cents

(7) Label each chord by letter with suffix above, and by Roman numeral with suffix below. Assume the major (Ionian) mode.

(a) $E_b\, ii$  
(b) $G_b\, vii$  
(c) $F\#\, vi$  

Suppose a musical tone is sounding $E_b$, and suppose its second harmonic has amplitude 3 and phase shift $\pi/4$. Express the second harmonic in the form $g(t) = A \sin(ct) + B \cos(ct)$, for explicit numbers $A$, $B$, and $c$.

2nd harmonic is $E_b^*$ with frequency $440 \cdot 2^{\frac{3}{440}} \sin(ict)$ has freq. $\frac{3}{2} \pi$, so $c = 2 \pi (440) \sqrt{2} \approx 3554.31$

$$g(t) = 3 \sin \left( 3554.31 t + \frac{\pi}{4} \right)$$

$$= 3 \left[ \cos \left( \frac{\pi}{4} \right) \sin \left( 3554.31 t \right) + \sin \left( \frac{\pi}{4} \right) \cos \left( 3554.31 t \right) \right]$$

$$= \frac{3\sqrt{2}}{2} \sin \left( 3554.31 t \right) + \frac{3\sqrt{2}}{2} \cos \left( 3554.31 t \right)$$
(9) On the staff below indicate the keyboard's best approximation of harmonics 2 through 11 of the given note.

(10) For each scale tone 1 to 8, write the fraction that expresses the ratio of that scale tone to 1 in the just intonation diatonic scale.

\[
\begin{array}{c|c}
1 & 1 \\
2 & \frac{11}{9} \\
3 & \frac{19}{15} \\
4 & \frac{22}{17} \\
5 & \frac{15}{11} \\
6 & \frac{2}{1} \\
7 & 2 \\
8 & 2 \\
\end{array}
\]

(11) Create the 5-tone row chart for the original row: [0] [2] [4] [3] [1], expressing each entry as an element of \( \mathbb{Z}_5 \) in the form \([r] \), where \( 0 \leq r < 5 \).
(12) Suppose the function \( y = f(t) \) is the periodic function of period \( P \) corresponding to a musical tone, and suppose the graph of \( y = f(t) \) is:

For each of the functions below, sketch its graph AND explain how its associated tone compares that of \( f(t) \).

(a) \( y = \frac{1}{2} f(t) \)

(b) \( y = f(2t) \)

(c) \( y = f(t) + c \)

(d) \( y = f(t - c) \)