

FINAL EXAM

Math 109 / Music 109A, Spring 2009

Name _____ Id _____

Each problem is worth 10 points.

1. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

(a) $f(x) = 2x^2 + 1$

(b) $g(x) = -1 + \sin \frac{3}{2}x$

2. For the set \mathbb{Z} and a fixed positive integer m , define the equivalence relation whose set of equivalence classes is \mathbb{Z}_m . Show that it is in fact an equivalence relation and explain why there are exactly m equivalence classes.

3. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef. Place the sharps and flats in their proper positions.

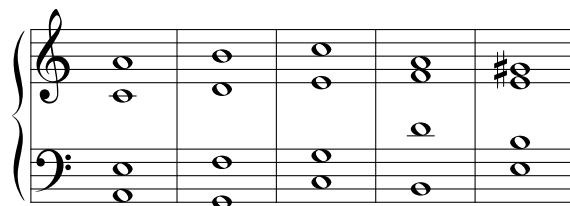
(a) Locrian with tonic D



(b) Myxolydian with tonic B^b



4. Identify each chord in this minor mode (Aeolian) passage. Above the staff label each chord by root note class with suffix (e.g., E^{b7}). Below the staff, label each chord by root scale tone (e.g. \flat III⁷).



5. (a) Use properties of logarithms to express in terms of logarithms of prime numbers:

$$\log_b \left[\left(\frac{20}{\sqrt[3]{21}} \right)^2 \right] =$$

- (b) Express as a single logarithm without coefficient, i.e., in the form $\log_b c$:

$$\log_b 15 - \frac{1}{2} \log_b 16 =$$

6. (a) Express the downward interval of 49 cents as ratio in three ways: as a power of 2, as a radical or the reciprocal of a radical, and by a decimal approximation.

(b) Convert the ratio $7/4$ to semitones, rounding off to 2 digits to the right of the decimal and indicating whether the interval is upward or downward. What is the name of this just interval?

7. Give the (total) duration in beats of:

(a) a sixteenth note in $\frac{12}{8}$ time (compound time signature).

(b) a dotted eighth note in $\frac{2}{2}$ time.

(c) a quarter note 5-tuplet in $\frac{4}{4}$ time.

8. Determine whether or not the pair $(\mathbb{Z}, +)$ is a monoid. If so, is it also also a group? Justify your answer.

9. Complete each excerpt with a measure which repeats the rhythm of the first measure, employing:

(a) diatonic transposition down one scale tone.



(b) chromatic transposition up a minor third.



10. Prove that if $y = f(t)$ has period P , then $y = cf(t)$ has period P , but $y = f(t/c)$ ($c \neq 0$) has period cP .

11. Find the period, frequency, amplitude, and phase shift for the function

$$g(t) = 3 \sin(440\pi t) + 4 \cos(440\pi t)$$

and express it in the form $d \sin(\alpha t + \beta)$, giving a decimal approximation for β .

12. The sawtooth wave, defined on $[0, 2\pi)$ by $q(t) = \frac{1}{\pi}t - 1$, has Fourier series

$$q(t) = -\frac{2}{\pi} \left[\sin t + \frac{1}{2} \sin(2t) + \frac{1}{3} \sin(3t) + \frac{1}{4} \sin(4t) + \cdots \right]$$

Give the values of the Fourier coefficients C, A_k, B_k for $k \in \mathbb{Z}^+$, indicate the phase shift of each harmonic, and explain why the value of C is such, based on the graph of $q(t)$.

13. The human “ah” vowel has a formant centered at 2640 Hz. A bass singer sings the vowel at G_2 . Which harmonics lie within 300 Hz of the center of this formant?

14. Express each of these just intervals in two ways: as rational numbers and in cents, rounding off the latter at 2 digits to the right of the decimal. What are their keyboard approximations, and how closely are they approximated?

(a) the lesser whole tone

(b) the just major third

15. Explain the comma of Pythagoras. How does it arise? Evaluate it as a rational number and in cents.