Homework 1
Math 109 / Music 109A, Spring 2006

Due Monday, January 30.

(1) For the following pairs of integers $m, n$, find the numbers $q$ and $r$ whose existence is asserted in the division algorithm:

(a) $17, 55$;

(b) $12, -37$;

(c) $2, 2^{21} + 3$;

(d) $7, 14k + 23$, where $k$ is some integer.

(2) Sketch the graphs of these functions, and indicate how each is obtained by geometric transformations (shifts and/or stretches) of simpler functions:

(a) $f(x) = \frac{1}{2}x - 1$

(b) $f(x) = x^2 + 1$

(c) $f(x) = 1 + \sin(2x)$

(3) For each of the following sets and relations determine whether or not an equivalence relation has been defined. Explain why or why not.

(a) The set of people alive now; “ has the same mother as”.

(b) $\mathbb{R}; \leq$.

(c) $\mathbb{Z}$; for a fixed positive integer $n$, $\equiv$ defined by $k \equiv \ell$ iff $n \mid k - \ell$.

(d) The set of keyboard note classes; $\sim$ defined by $N \sim N'$ iff the interval between $N$ and $N'$ is either the unison interval or a major third (up or down).

(4) For the set $\{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\}$ show that the relation $\sim$ defined by $(a, b) \sim (a', b')$ iff $ab' - a'b = 0$ is an equivalence relation and that the set of equivalence classes is in one-to-one correspondence with $\mathbb{Q}$.
(5) Identify these notes by letter and subscript (e.g., D₃ or A₄):

(a) \[ \text{Note A} \]
(b) \[ \text{Note B} \]
(c) \[ \text{Note C} \]
(d) \[ \text{Note D} \]

(6) Identify these intervals:

(a) \[ \text{Interval E} \]
(b) \[ \text{Interval F} \]
(c) \[ \text{Interval G} \]
(d) \[ \text{Interval H} \]

(7) Write on staff paper, and name with subscript, the note which is:

(a) a minor third above D₂.
(b) a fifth above F₃.
(c) a major ninth below C₆.
(d) a tritone below E₄.

(8) Notate all the key signatures on a line of staff paper. Order the keys using flats by ascending number of flats and similarly for those using sharps. For each indicate the major key and the minor key it denotes.

(9) For the following modes and tonic notes, indicate the appropriate key signature on staff paper:

(a) Lydian with tonic G.
(b) Dorian with tonic B♭.
(c) Locrian with tonic D♭.
(d) Phrygian with tonic A.

(10) Transpose this melodic excerpt, written in C minor, up to E minor. Preserve the scale-tone spelling of each melody note.