

# Homework 3

Math 109 / Music 109A, Spring 2009

Due Monday, February 23.

1. Express each of these intervals as elements of  $\mathbb{R}^+$  three ways: (1) as a radical or the reciprocal of a radical, (2) as a power of 2, and (3) by a decimal approximation with 3 decimal digits.

- (a) up 47 cents
- (b) down 325 cents
- (c) up a minor sixth
- (d) the interval from  $G_3$  to  $A_1^\sharp$

2. Assuming  $A_4$  is tuned to 440 Hz, find the frequencies for:

- (a)  $C_4$
- (b)  $E_2^\flat$
- (c)  $B_5$
- (d)  $A_3^\flat$

Suppose middle C is tuned as 256 Hz. Find the frequencies for:

- (a)  $A_4$
- (b)  $F_5^\flat$
- (c)  $C_1$
- (d)  $G_2^\sharp$

3. For each of these chords, voiced within an octave with the root on the bottom, give the pitch of each note in the chord. Assume  $A_4$  is tuned to 440 Hz.

- (a) major triad with root  $G_3$
- (b) minor triad with root  $A_4^\flat$
- (c) minor seventh chord with root  $D_3$
- (d) diminished seventh with root  $B_5^\flat$

4. Suppose a string on a banjo has length 50cm. Indicate positions of the 12 frets which will allow the string to play one octave of the ascending chromatic scale.

5. Evaluate without a calculator by writing the argument of log as a power of the base. Write down each step of the simplification, e.g.,  $\log_3 3\sqrt{3} = \log_3 3^{3/2} = \frac{3}{2} \log_3 3 = \frac{3}{2}$ :

(a)  $\log_{10}(0.0001)$     (b)  $\log_3 243$     (c)  $\log_5 \sqrt[7]{25}$     (d)  $\log_b(1/\sqrt[n]{b^\ell})$

Express as a single logarithm without coefficient, i.e., in the form  $\log_b c$  (do not evaluate with a calculator):

(e)  $\log_3 11 + \log_3 17$     (f)  $\log_9 5 - 2 \log_9 2$   
(g)  $\log_2 13 + \log_4 21$     (h)  $2 \log_c x^2 - \frac{1}{2} \log_{\sqrt{c}} x$

6. Sketch the graphs of:

(a)  $f(x) = 2^x$     (b)  $g(x) = \log_2 x$     (c)  $r(x) = 5^x$     (d)  $s(x) = \log_5 x$

Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.

7. Using laws of exponents, prove this property of logarithms:

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

8. Convert to semitones the intervals given by the following ratios: (Round off to 2 digits to the right of the decimal.)

(a) 7    (b) 0.3    (c)  $\frac{5}{2}$     (d)  $\sqrt[4]{4}$     (e)  $e$

Convert to cents the intervals given by the following ratios, rounding off to the nearest whole cent:

(f) 0.8    (g) 1.2    (h)  $\frac{3}{5}$     (i)  $\frac{8}{3}$     (j)  $\pi$

9. Give a plausible harmonization of this melody by providing, in the bass clef, one whole note chord for each measure. Label each chord by root scale tone (Roman numeral) and chord type (e.g., II<sup>m</sup>7).

The image shows a musical score for a melody in 4/4 time, starting with a treble clef and a key signature of one flat (Bb). The melody consists of four measures. The first measure has notes Bb4, C5, D5, E5. The second measure has notes F#5, G5, A5, B5. The third measure has notes C6, B5, A5, G5. The fourth measure has notes F5, E5, D5, C5. The bass clef is empty for all measures.

10. Analyze the basic harmony in the first 16 measures of *Maple Leaf Rag*. Each measure will have at most two chords. Label the chords by root note class and chord type (e.g., G<sup>7</sup>). (Note: In a few places the chords are incomplete.) The music can be downloaded as a pdf file from the website.