Homework 6

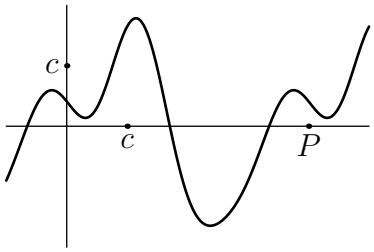
Math 109 / Music 109A, Spring 2009

Due Monday, April 13.

1. Prove that if $y = f(t)$ has period P, then $y = f(t/c)$ $(c \neq 0)$ has period cP.

2. If $y = f(t)$ has period P, and $a_1, \ldots, a_n$ are any real coefficients, prove that
$$h(t) = \sum_{k=1}^{n} a_k f(kt)$$
also has period P.

3. Suppose the function $y = f(t)$ is the periodic function of period P corresponding to a musical tone, and suppose the graph of $y = f(t)$ is:



For each of the functions below, sketch its graph and explain how its associated tone compares that of $f(t)$.

(a) $y = \frac{1}{2} f(t)$ (b) $y = f(2t)$

(c) $y = f(t) + c$ (d) $y = f(t + c)$

4. Find the value α for which the pitch associated to the periodic function $y = \sin(\alpha t)$, where t is time in seconds, is:

(a) middle C (b) $\mathrm{A}_{\flat}^{2}$ (c) $\mathrm{D}_{\sharp}^{6}$

5. Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form $A\sin(\alpha t) + B\cos(\alpha t)$:

   (a) $f(t) = 5\sin(30\pi t + \frac{\pi}{4})$

   (b) $g(t) = \sqrt{2}\sin(800t + \pi)$

6. Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form $d\sin(\alpha t + \beta)$:

   (a) $f(t) = 4\sin(300t) + 5\cos(300t)$

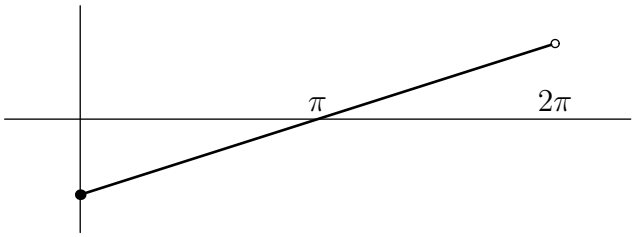
   (b) $h(t) = -\sin(1500\pi t) + 3\cos(1500\pi t)$

7. Suppose musical tone with pitch B_4 has harmonics 1, 3, 5 only, with amplitudes 1, $\frac{1}{9}$, $\frac{1}{25}$, respectively, and phase shifts 0, π, $-\frac{\pi}{2}$, respectively. Suppose also that the vertical shift C is 0. Write its Fourier series in the form $\sum[A_k\sin(kt) + B_k\cos(kt)]$.

8. The human *ee* vowel has a formant centered at 2900 Hz. What pitch should one sing in order for the fifth harmonic to be maximally amplified by this formant?

9. Two instruments play the pitches A_2 and E_3, making the interval of a keyboard fifth. Suppose they are playing the same kind of instrument, and that the instrument has a formant centered at 3000 Hz. Suppose the formant amplifies pitches within 400 Hz of its center. Identify the harmonics produced by each instrument which will be amplified by the formant, and give their frequencies. How many pairs of these frequencies are almost aligned? Could this "near alignment" be perfected by slightly adjusting the interval?

10. Let $q(x)$ be defined by $q(x) = \frac{1}{\pi}x - 1$ on the interval $[0, 2\pi)$, extended to a periodic function on $\mathbb{R}$ by periodicity. This a *sawtooth wave.* It's graph on $[0, 2\pi)$ is:



$y = q(x)$

The sound of this waveform is a harsh buzz. It's Fourier series is

$$q(x) = -\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{k}\sin(kt)\,.$$

For $k = 1$ verify that this formula gives the correct sine coefficient. Hint: Mimic the computation for the square wave. You will need the formula

$$\int_0^{2\pi} t\sin(kt)\,dt = -\frac{2\pi}{k}$$

(which calculus students can verify using integration by parts).