Tue 9

Name: Solutions ID Number: ______

1. The amount of a product produced is given by

$$N(x,y) = 24x^{3/4}y^{1/4}$$

with x and y the number of units of labor and capital respectively. If each unit of labor costs \$50 and each unit of capital costs \$100 and the total budget is \$600,000:

3 points **a.** What allocation of money to labor and capital maximizes the amount of product produced?

$g(x,y) = 50x + 100y - 600{,}000$

$F(x,y,\lambda) = 24x^{3/4}y^{1/4} + \lambda(50x + 100y - 600{,}000)$

$F_x = 18x^{-\frac{1}{4}}y^{\frac{1}{4}} + 50\lambda = 0$

$F_y = 6x^{3/4}y^{-3/4} + 100\lambda = 0$

$F_\lambda = 50x + 100y - 600{,}000 = 0$

$\lambda = -\frac{9}{25}x^{-\frac{1}{4}}y^{\frac{1}{4}}$

$\lambda = -\frac{3}{50}x^{3/4}y^{-3/4}$

These are equal!

$\left[-\frac{9}{25}x^{-\frac{1}{4}}y^{\frac{1}{4}} = -\frac{3}{50}x^{3/4}y^{-3/4}\right](x^{\frac{1}{4}} \cdot y^{3/4} \cdot 50)(-1)$

$18y = 3x$, so, $x = 6y$ substitute into

$F_\lambda = 0$:

$50(6y) + 100y = 600{,}000$

$400y = 600{,}000$

$y = 1500$

$x = 9000$

$\lambda = -\frac{9}{25}(9000)^{-\frac{1}{4}}(1500)^{1/4} \cdot -\frac{9}{25}\frac{1500^{1/4}}{(1500)^{1/4}(6)^{1/4}}$

$= -.2300145175$

1 point **b.** What is that maximum amount of product?

$N(9000, 1500) = 24(9000)^{3/4}(1500)^{1/4} = 24(6)^{3/4}(1500) = (36{,}000)\,6^{3/4}$

$= 138011.7105$

2 points **c.** At that maximum situation what is the numerical value of

i. the marginal utility of labor? $N_x = 18x^{-\frac{1}{4}}y^{\frac{1}{4}}$, so $N_x(9000, 1500) = \frac{18}{(9000)^{1/4}} \cdot (1500)^{1/4}$

$N_x(9000, 1500) = \frac{18}{6^{1/4}} \approx 11.50072588$

ii. the marginal utility of capital? $N_y = 6x^{3/4}y^{-3/4}$

$N_y(9000, 1500) = \frac{6(9000)^{3/4}}{(1500)^{3/4}} = 6 \cdot 6^{3/4} \approx 23.00145175$

iii. the marginal utility of money? $(-\lambda) = .2300145175$

Tue 10

Name: Solutions ID Number:

1. Use Lagrange multipliers to find the minimum of $x^2 + y^2$ subject to the constraint $x + 2y = 12$.

$f(x,y) = x^2+y^2$

$g(x,y) = x+2y-12=0,$

$F(x,y,\lambda) = x^2+y^2+\lambda(x+2y-12)$

$F_x = 2x + \lambda = 0, \quad \lambda = -2x$

$F_y = 2y + 2\lambda = 0, \quad \lambda = -y$

$2x = y$

$F_\lambda = x+2y-12 = 0$

substitute

$x + 4x - 12 = 0$

$x = \frac{12}{5}$

$y = \frac{24}{5}$

Minimum:

$f\left(\frac{12}{5}, \frac{24}{5}\right) = \frac{144}{25} + \frac{4(144)}{25}$

$= \frac{5(144)}{25}$

$= \frac{144}{5} = 28.8$

2. Use Lagrange multipliers to find the maximum of $25 - 2x^2 - y^2$ subject to the constraint $x + 2y = 4$.

$f(x,y) = 25 - 2x^2 - y^2$

$g(x,y) = x+2y-4$

$F(x,y,\lambda) = 25-2x^2-y^2+\lambda(x+2y-4)$

$F_x = -4x + \lambda = 0, \quad \lambda = 4x$

$F_y = -2y + 2\lambda, \quad \lambda = y$

$y = 4x$

$F_\lambda = x+2y-4=0$

substitute

$x + 8x - 4 = 0$

$x = \frac{4}{9}$

$y = \frac{16}{9}$

Maximum: $f\left(\frac{4}{9}, \frac{16}{9}\right)$

$= 25 - 2\left(\frac{16}{81}\right) - \frac{(16)^2}{81}$

$= 25 - \frac{288}{81} = 21.444\ldots$

Tue 11

Name: Solutions ID Number:

1. Use Lagrange multipliers to find the minimum of $2x^2 + y^2$ subject to the constraint $x + 2y = 12$.

$f(x,y) = 2x^2 + y^2$

$g(x,y) = x + 2y - 12 = 0$

$F(x,y,\lambda) = 2x^2 + y^2 + \lambda(x + 2y - 12)$

$F_x = 4x + \lambda = 0 \quad \lambda = -4x$ $\quad y = 4x$

$F_y = 2y + 2\lambda \quad \lambda = -y$

$F_\lambda = x + 2y - 12 = 0$ ← substitute

$x + 8x - 12 = 0$

$x = \frac{12}{9} = \frac{4}{3}$

$y = \frac{16}{3}$

Min: $f\left(\frac{4}{3}, \frac{16}{3}\right) = 2\left(\frac{16}{9}\right) + \frac{256}{9}$

$= \frac{288}{9} = 32$

2. Use Lagrange multipliers to find the maximum of $25 - x^2 - y^2$ subject to the constraint $x + y = 4$.

$f(x,y) = 25 - x^2 - y^2$

$g(x,y) = x + y - 4 = 0$

$F(x,y,\lambda) = 25 - x^2 - y^2 + \lambda(x + y - 4)$

$F_x = -2x + \lambda = 0, \quad \lambda = 2x$ $\quad x = y$

$F_y = -2y + \lambda = 0, \quad \lambda = 2y$

$F_\lambda = x + y - 4 = 0$ ← substitute

$x + x - 4 = 0$

$x = 2,$

$y = 2$

Max. $f(2,2) = 25 - 4 - 4$

$= 17$