Calculus II for the Life, Social and Managerial Sciences

Math 128 — Fall 2007

In-term exam September 26

Name: **Student-ID:**

This exam contains sixteen problems. Problems 1 – 14 are multiple choice problems, which each count 5% towards your total score. Problem 15 and 16 will be hand-graded (with a possibility of partial credit) and count 15% each towards your total score.

Problem 1

$\int (e^{2x} + 4x^3 + 7)\,dx =$ $\frac{1}{2} e^{2x} + x^4 + 7x + C$

A) $\frac{1}{2}e^{2x} + x^4 + C$

B) $\frac{1}{2}e^{2x} + 4x^4 + 7x + C$

C) $\frac{1}{2}e^{2x} + x^4 + 7x + C$

D) $e^{2x} + x^4 + 7x + C$

E) $e^{2x} + 4x^4 + 7x + C$

F) $e^{2x} + x^4 + 7 + C$

G) $2e^{2x} + 12x^2 + C$

H) $2e^{2x} + x^4 + 7x + C$

Problem 2

Suppose that the rate (in tons per year) at which pollutants are discharged into a lake is given by

$$R(t) = \frac{1000}{(1+t)^2}$$

where t is the time in years since January 1, 2000. Find the total amount of discharged pollutants (in tons) between January 1, 2000 and January 1, 2004.

$$\text{Total} = \int_0^4 \frac{1000}{(1+t)^2}\,dt = 1000\left(-\frac{1}{1+t}\right)_0^4$$

$$= 1000\left[-\frac{1}{5} + 1\right]$$

$$= 1000\left(\frac{4}{5}\right) = 800$$

A) 300

B) 400

C) 500

D) 600

E) 700

F) 800

G) 900

H) 1000

Problem 3

Find the area under the curve $y = x^9 + 26x + 7$ from $x = 0$ to $x = 1$.

$$\int_0^1 x^9 + 26x + 7\,dx = \frac{x^{10}}{10} + 13x^2 + 7x \Big|_0^1$$

$$= \frac{1}{10} + 13 + 7 = 20.1$$

A) 20.1

B) 20.2

C) 20.3

D) 20.4

E) 20.5

F) 20.6

G) 20.7

H) 20.8

Problem 4

Determine the average value of $f(x) = 200e^{-4x}$ over the interval from $x = 0$ to $x = 2$.

A) $25 - 25\,e^{-4}$

B) $25 - 25\,e^{-8}$

C) $50 - 50\,e^{-4}$

D) $50 - 50\,e^{-8}$

E) $100 - 100\,e^{-4}$

F) $100 - 100\,e^{-8}$

G) $200 - 200\,e^{-4}$

H) $200 - 200\,e^{-8}$

$$\frac{1}{2-0}\int_0^2 200\,e^{-4y}\,dy$$

$$= \frac{1}{2}\cdot 200\left(-\frac{1}{4}\right)\left[e^{-4y}\right]_0^2$$

$$= -25\left[e^{-8}-1\right]$$

$$= 25 - 25e^{-8}$$

Problem 5

Find the consumers' surplus for the demand curve $p = \frac{4000}{(x+10)^2} - 3$ at the sales level $x = 10$.

A) 50

B) 100

C) 150

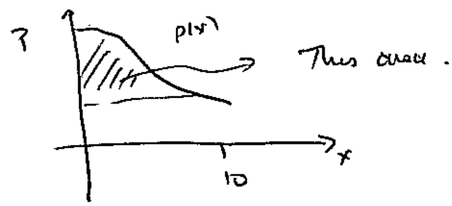
D) 200

E) 250

F) 300

G) 350

H) 400



$$p(10) = \frac{4000}{(20)^2} - 3 = 10 - 3 = 7.$$

$$\int_0^{10} \frac{4000}{(x+10)^2} - 3 - 7 \, dx.$$

$$= 4000 \int_0^{10} \frac{1}{(x+10)^2} dx - 100.$$

$$= 4000 \left(-\frac{1}{(x+10)} \right]_0^{10} - 100$$

$$= 4000 \left(-\frac{1}{20} + \frac{1}{10} \right] - 100$$

$$= 4000 \left(\frac{1}{20} \right) - 100$$

$$= 100.$$

Problem 6

Suppose that money is deposited daily into a savings account at an annual rate of \$2000. If the account pays 5% interest compounded continuously, estimate (to the nearest dollar) the balance in the account at the end of 4 years.

A) \$8856
B) \$8861
C) \$8868
D) \$8875
E) \$8882
F) \$8889
G) \$8896
H) \$8903

Future value is:

$$2000\int_0^4 e^{\frac{1}{20}(4-t)}\,dt.$$

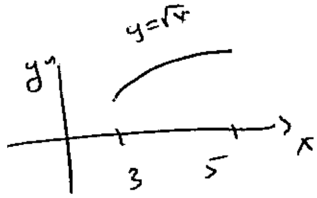
$$= 2000\,(-20)\left[e^{\frac{1}{20}(4-t)}\right]_0^4$$

$$= -40{,}000\left[1 - e^{\frac{1}{5}}\right].$$

$$\simeq 8856.$$

Problem 7

Find the volume of the solid of revolution obtained from revolving the region below the graph of $y = \sqrt{x}$ from $x = 3$ to $x = 5$ about the x-axis.



A) π

B) 2π

C) 3π

D) 4π

E) 5π

F) 6π

G) 7π

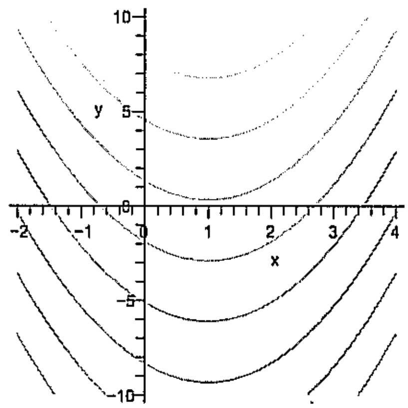
H) 8π

$$\pi \int_3^5 x\,dx = \pi\left[\frac{x^2}{2}\right]_3^5$$

$$= \frac{\pi}{2}\left[25 - 9\right] = 8\pi$$

Problem 8

Which function $f(x, y)$ do these level curves belong to?



These are the curves:

$y = (x-1)^2 + C.$

C a constant,

So, $C = y - (x-1)^2$

$Z = y - (x-1)^2.$

A) $f(x, y) = y + (x-1)^2$

B) $f(x, y) = (x-1)^2 + y^2$

C) $f(x, y) = 2(x-1)^2 + y^2$

D) $f(x, y) = y + x - 1$

E) $f(x, y) = (x-1)^2 + 2y^2$

F) $f(x, y) = y - (x-1)^2$

G) $f(x, y) = y - (x+1)^2$

H) $f(x, y) = y - x^2$

Problem 9

Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function

$$f(x, y) = e^{3x} + 5(x - y)^3$$

Then $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = \cdots$

$\frac{\partial f}{\partial x} = 3e^{3x} + 3 \cdot 5 (x-y)^2$

A) $e^{3x} + 15(x - y)^2$

B) $3e^{3x} + 30(x - y)^2$

C) $3xe^{3x} + 30(x - y)^2$

D) $3e^{3x}$

E) e^{3x}

F) $e^{3x} + 30(x - y)^2$

G) $3e^{3x} + 15(x - y)^2$

H) $3e^{3x} - 30(x - y)^2$

$\frac{\partial f}{\partial y} = - 15 (x-y)^2.$

$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 3e^{3x}$

Problem 10

Let $f(x, y, z) = x^2y - 3xz + 6y^3z + 2y - 7z^2 + 37$.
Evaluate $\frac{\partial f}{\partial z}$ at the point $(14, 1, -3)$.

$\frac{\partial f}{\partial z} = -3x + 6y^3 - 14z.$

at $(14,1,-3)$ this is:

$-3(14) + 6 + 3(14)$

$= 6$

A) 1

B) -3

C) 6

D) 2

E) -7

F) 37

G) 14

H) -78

Problem 11

Compute the second partial derivatives $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ of the function

$$f(x, y) = xe^y + 2x^2y^2$$

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \cdots$

$\frac{\partial f}{\partial x} = e^y + 4xy^2$

$\frac{\partial^2 f}{\partial x^2} = 4y^2$

$\frac{\partial f}{\partial y} = xe^y + 4x^2y$

$\frac{\partial^2 f}{\partial y^2} = xe^y + 4x^2.$

A) $xe^y + 4y^2$

B) xe^y

C) $4y^2$

D) $xe^y + 4x^2$

E) $xe^y + 4x^2 + 4y^2$

F) $4x^2 + 4y^2$

G) $4x^2$

H) 0

Problem 12

Using Lagrange multipliers, minimize $f(x,y) = (x-1)^2 + y^2$, subject to the constraint $x + 2y - 11 = 0$.

At which point (x,y) does this minimum occur?

A) $(3,4)$
B) $(3,-4)$
C) $(-4,3)$
D) $(-4,-3)$
E) $(-3,-4)$
F) $(-3,4)$
G) $(4,-3)$
H) $(4,3)$

$F(x,y,\lambda) = (x-1)^2 + y^2 + \lambda(x+2y-11)$

$\partial_x F = 2(x-1) + \lambda = 0 \qquad \lambda = -(2)(x-1)$

$\qquad\qquad\qquad\qquad\qquad\qquad = -y.$

$\partial_y F = 2y + 2\lambda = 0$

$\partial_\lambda F = x + 2y = 11 \qquad$ So $y = 2(x-1)$.

$x + 2(2)(x-1) - 11 = 0$

$5x - 4 = 11$

$5x = 15$

$x = 3. \quad y = 4$

Problem 13

Use partial derivatives to obtain the formula for the best least-squares fit to the data points $(1, 8)$, $(2, 4)$, $(4, 3)$.

This line is given by which formula?

A) $y = -0.5x + 8.5$

B) $y = -x + 7.5$

C) $y = -x + 8.5$

D) $y = -1.5x + 7.5$

E) $y = -1.5x + 7$

F) $y = -x + 7$

G) $y = -1.5x + 8.5$

H) $y = -0.5x + 8$

$N = 3$

$\sum xy = 8 + 8 + 12 = 28$

$\sum x = 7$

$\sum y = 15$

$\sum x^2 = 21$

$$A = \frac{3(28) - (7)(15)}{3(21) - 7^2} = \frac{\cancel{(7)}(3)\left[4 - 5\right]}{\cancel{(7)}\left[9 - 7\right]} = \frac{-3}{2}$$

$$B = \frac{15 - \left(-\frac{3}{2}\right)7}{3} = \frac{15 + \frac{21}{2}}{3} = \frac{51}{6} = 8.5$$

$y = -1.5x + 8.5$

Problem 14

Evaluate the iterated integral

$$\int_0^1 \left(\int_1^2 (6xy + 2)\, dy \right) dx$$

A) 4

B) 4.5

C) 5

D) 5.5

E) 6

F) 6.5

G) 7

H) 7.5

$$\int_0^1 \left[3xy^2 + 2y \right]_1^2 dx$$

$$= \int_0^1 \left[12x + 4 - (3x + 2) \right] dx$$

$$= \int_0^1 (9x + 2)\, dx$$

$$\frac{9}{2}x^2 + 2x \Big|_0^1 = 6.5.$$

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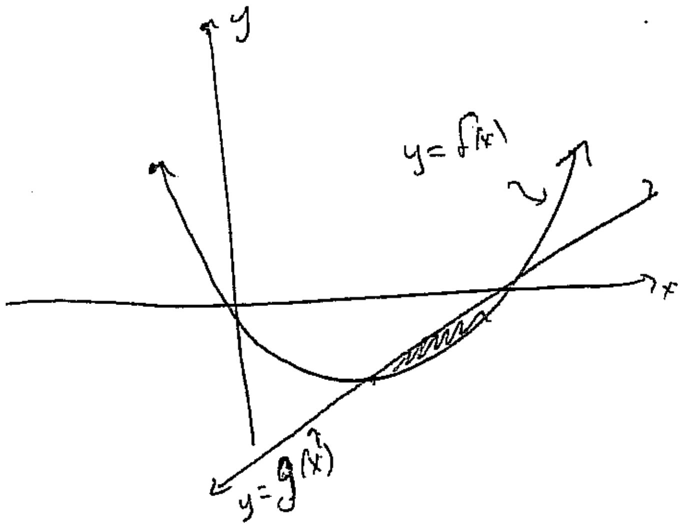
Written problem - Show your work

Problem 15

The graphs of $f(x) = x^2 - 4x$ and $g(x) = 2x - 8$ intersect at the points $(2, -4)$ and $(4, 0)$. Calculate the area between these two graphs.



$f(3) = -3$

$g(3) = -2$

So, $g(x) > f(x)$ on $(2, 4)$

$$\text{Area} = \int_2^4 2x - 8 - (x^2 - 4x)\,dx = \int_2^4 6x - 8 - x^2\,dx$$

$$= 3x^2 - 8x - \frac{x^3}{3} \Big|_2^4 = \left[48 - 32 - \frac{64}{3} - \left(12 - 16 - \frac{8}{3}\right)\right].$$

$$= 16 - \frac{64}{3} - \left(-4 - \frac{8}{3}\right)$$

$$= 20 - \frac{64}{3} + \frac{8}{3} = 20 - \frac{56}{3}$$

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Written problem - Show your work

Problem 16

Find all points where $f(x, y) = x^3 + y^2 - 3x - 8y + 12$ has a possible extreme value. Then, using the second-derivative test, determine for each of these points its nature (relative maximum, relative minimum, or saddle point).

$\frac{\partial f}{\partial x} = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

$\frac{\partial f}{\partial y} = 2y - 8 \Rightarrow y = 4.$

Critical Points: $(1,4)$, $(-1,4)$.

$\frac{\partial^2 f}{\partial x^2} = 6x$, $\frac{\partial^2 f}{\partial y^2} = 2.$ $\frac{\partial^2 f}{\partial x \partial y} = 0.$

$D(x,y) = 12x$

at $(1,4)$ $D(1,4) = 12 > 0.$ $\frac{\partial^2 f}{\partial x^2} = 6 > 0 \Rightarrow$ min

at $(-1,4)$ $D(-1,4) = -12 < 0 \Rightarrow$. a saddle point at $(-1,4)$