Calculus II for the Life, Social and Managerial Sciences

Math 128 — Fall 2007

In-term exam October 24

Name: **Student-ID:**

This exam contains sixteen problems. Problems 1 – 14 are multiple choice problems, which each count 5% towards your total score. Problem 15 and 16 will be hand-graded (with a possibility of partial credit) and count 15% each towards your total score.

Problem 1

Solve the equation $\cos t = \sin t$ for $0 \leq t \leq \frac{3\pi}{2}$.

$\tan(t) = 1$,
$t = \pi/4, \frac{5\pi}{4}$

A) $t = \frac{\pi}{4}$
B) $t = \frac{3\pi}{4}$
C) $t = \frac{5\pi}{4}$
D) $t = \frac{7\pi}{4}$
E) $t = \frac{\pi}{4}$ or $t = \frac{5\pi}{4}$
F) $t = \frac{3\pi}{4}$ or $t = \frac{7\pi}{4}$
G) $t = \frac{\pi}{4}$ or $t = \frac{3\pi}{4}$
H) $t = \frac{3\pi}{4}$ or $t = \frac{5\pi}{4}$

Problem 2

Compute

$$\int_0^{\pi/17} 5\sin(17x-\pi)\,dx$$

$u = 17x - \pi$

$du = 17dx$

$$\frac{5}{17}\int_{-\pi}^{0} \sin(u)du$$

$= -\frac{5}{17} \cos(u) \Big|_{-\pi}^{0}$

$= \frac{-5}{17}\left[1 - -1\right].$

$= \frac{-10}{17}.$

A) $-\frac{10}{17}$

B) $-\frac{5}{17}$

C) $\frac{5}{17}$

D) $\frac{10}{17}$

E) $-\frac{10}{\pi}$

F) $-\frac{5}{\pi}$

G) $\frac{5}{\pi}$

H) $\frac{10}{\pi}$

Problem 3

Let $f(x) = \tan(\sqrt{x-1})$. Calculate $f'(x)$.

A) $\frac{1}{\cos^2(\sqrt{x-1})}$

B) $\frac{2\sqrt{x-1}}{\cos^2(\sqrt{x-1})}$

C) $\tan(\frac{1}{2\sqrt{x-1}})$

D) $\frac{1}{2\sqrt{x-1}\tan^2(\sqrt{x-1})}$

E) $\frac{1}{\sqrt{x-1}\cos^2(\sqrt{x-1})}$

F) $\frac{\sqrt{x-1}}{\cos^2(\sqrt{x-1})}$

G) $\frac{\sqrt{x-1}}{\tan^2(\sqrt{x-1})}$

H) $\frac{1}{2\sqrt{x-1}\cos^2(\sqrt{x-1})}$

$$f'(x) = \frac{1}{2}\frac{\sec^2(\sqrt{x-1})}{\sqrt{x-1}}$$

$$= \frac{1}{(2\sqrt{x-1})\cos^2(\sqrt{x-1})}$$

Problem 4

Evaluate the definite integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{12}} \frac{1}{\cos^2(3x)}\,dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{5\pi}{12}} \sec^2(3x)\,dx$$

$u = 3x$

$du = 3dx$

$$= \frac{1}{3}\int_{\pi}^{\frac{5\pi}{4}} \sec^2(u)\,du$$

$$= \frac{1}{3}\left[\tan(u)\right]_{\pi}^{5\pi/4}$$

$$= \frac{1}{3}\left[1 - 0\right].$$

$$= \frac{1}{3}$$

A) $-\frac{1}{3}\sqrt{3}$

B) $-\frac{1}{3}$

C) $-\frac{1}{9}\sqrt{3}$

D) 0

E) $\frac{1}{9}\sqrt{3}$

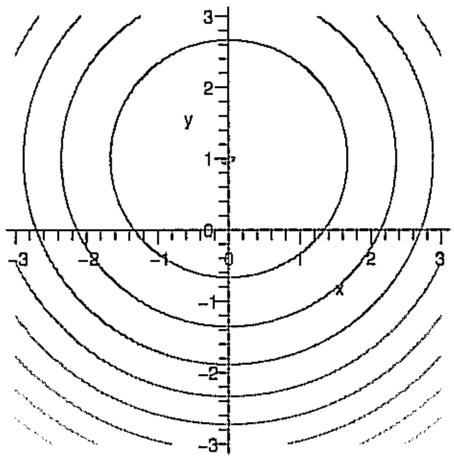
F) $\frac{1}{3}$

G) $\frac{1}{3}\sqrt{3}$

H) 1

Problem 5

Which function $f(x, y)$ do these level curves belong to?



Circle centered at $(0,1)$

has equation

$x^2 + (y-1)^2 = r^2$

Radius, changing

So, $z = (x)^2 + (y-1)^2$.

A) $f(x, y) = x^2 + (y + 1)^2$

B) $f(x, y) = (x - 1)^2 + y^2$

C) $f(x, y) = (x + 1)^2 + y^2$

D) $f(x, y) = (x - 1)^2 + (y - 1)^2$

E) $f(x, y) = (x + 1)^2 + (y - 1)^2$

F) $f(x, y) = (x - 1)^2 + 2y^2$

G) $f(x, y) = x^2 + (y - 1)^2$

H) $f(x, y) = x^2 + 2(y - 1)^2$

Problem 6

Compute the indefinite integral

$$\int \frac{e^x}{(1+e^x)^2}\,dx$$

$u = 1+e^x$

$du = e^x$

$\int \frac{du}{u^2} = -\frac{1}{u} + C$

$= -\frac{1}{1+e^x} + C.$

A) $\frac{1}{2} \cdot \frac{1}{1+e^x} + C$

B) $-\frac{1}{2} \cdot \frac{1}{1+e^x} + C$

C) $\frac{1}{1+e^x} + C$

D) $-\frac{1}{1+e^x} + C$

E) $\frac{1}{2} \cdot \frac{e^x}{1+e^x} + C$

F) $-\frac{1}{2} \cdot \frac{e^x}{1+e^x} + C$

G) $\frac{1}{(1+e^x)^2} + C$

H) $-\frac{1}{(1+e^x)^2} + C$

Problem 7

Evaluate

$$\int x\sqrt{2-x}\,dx$$

A) $\frac{2}{3}x(2-x)\sqrt{2-x} - \frac{2}{3}(2-x)\sqrt{2-x} + C$

B) $-\frac{2}{3}x(2-x)\sqrt{2-x} + \frac{2}{3}(2-x)\sqrt{2-x} + C$

C) $\frac{2}{3}x(2-x)\sqrt{2-x} - \frac{5}{3}(2-x)\sqrt{2-x} + C$

D) $-\frac{2}{3}x(2-x)\sqrt{2-x} + \frac{5}{3}(2-x)\sqrt{2-x} + C$

E) $\frac{2}{3}x(2-x)\sqrt{2-x} + \frac{4}{15}(2-x)^2\sqrt{2-x} + C$

F) $-\frac{2}{3}x(2-x)\sqrt{2-x} + \frac{4}{15}(2-x)^2\sqrt{2-x} + C$

G) $-\frac{2}{3}x(2-x)\sqrt{2-x} - \frac{4}{15}(2-x)^2\sqrt{2-x} + C$

H) $\frac{2}{3}x(2-x)\sqrt{2-x} - \frac{4}{15}(2-x)^2\sqrt{2-x} + C$

Integrate By Parts

$u = x \qquad dv = \sqrt{2-x}$

$du = dx \qquad v = -\frac{2}{3}(2-x)^{3/2}$

$$\int x\sqrt{2-x}\,dx = -\frac{2}{3}x(2-x)^{3/2} + \frac{2}{3}\int (2-x)^{3/2}\,dv$$

$$= -\frac{2}{3}x(2-x)^{3/2} \;\cancel{\;}\; \frac{4}{15}(2-x)^{5/2} + C$$

Problem 8

Find the area under the curve $y = x^2 e^x$ from $x = 0$ to $x = 2$.

A) $2e^2 - 2$

B) $2e^2 + 2$

C) $2e^2 - 4$

D) $2e^2 + 4$

E) $4e^2 + 4$

F) $4e^2 - 4$

G) $4e^2 + 2$

H) $4e^2 - 2$

$$\int x^2 e^x dx$$

$u = x^2 \quad dv = e^x$

$du = 2x dx \quad v = e^x$

$$= x^2 e^x - \int 2x e^x dx$$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$= x^2 e^x - 2[xe^x - e^x]$$

$$\int_0^2 x^2 e^x = x^2 e^x - 2xe^x + 2e^x \Big|_0^2$$

$$= 4e^2 - 4e^2 + 2e^2$$

$$- (2)$$

$$= 2e^2 - 2$$

Problem 9

Determine the average value of $f(x) = \frac{1}{x}(\ln x)^2$ over the interval from $x = 1$ to $x = 3$.

A) $\frac{1}{6}(\ln 2)^3$

B) $\frac{1}{6}(\ln 3)^3$

C) $\frac{1}{4}(\ln 2)^2$

D) $\frac{1}{3}(\ln 3)^3$

E) $\frac{1}{2}(\ln 2)^2$

F) $\frac{1}{2}(\ln 3)^3$

G) $(\ln 2)^2$

H) $(\ln 3)^3$

$$\frac{1}{2} \int_1^3 \frac{1}{x} (\ln|x|)^2 \, dx$$

$$\left(\begin{array}{l} u = \ln|x| \\ du = \frac{1}{x}\,dx \end{array} \right)$$

$$\frac{1}{2} \left. \frac{(\ln(x))^3}{3} \right|_1^3$$

$$= \frac{1}{6} \left[\ln(3)^3 \right].$$

Problem 10

Use the Midpoint Rule with $n = 4$ to approximate $\int_1^5 \frac{1}{\ln(x+0.5)}\,dx$ to 4 decimal places.

midpoints are:

midpoints	$f(x)$
1.5,	$\frac{1}{\ln(2)}$
2.5	$\frac{1}{\ln(3)}$
3.5	$\frac{1}{\ln(4)}$
4.5	$\frac{1}{\ln(5)}$

A) 3.6710

B) 3.6833

C) 3.6956

D) 3.7079

E) 3.7202

F) 3.7325

G) 3.7448

H) 3.7571

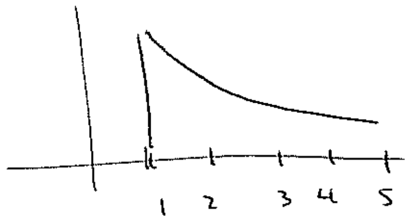
$f(x) = \frac{1}{\ln(x+.5)}$ $\Delta x = 1$

$$\int_1^5 \frac{1}{\ln(x+.5)}\,dx \approx 1\left(\frac{1}{\ln(2)} + \frac{1}{\ln(3)} + \frac{1}{\ln(4)} + \frac{1}{\ln(5)}\right)$$

≈ 3.6956

Problem 11

Use the Trapezoidal Rule with $n = 4$ to approximate $\int_1^5 \frac{1}{\ln(x+0.5)}\,dx$ to 4 decimal places.



A) 4.0563

B) 4.0686

C) 4.0809

D) 4.0932

E) 4.1055

F) 4.1178

G) 4.1301

H) 4.1424

x	$f(x)$
1	$\frac{1}{\ln(1.5)}$
2	$\frac{1}{\ln(2.5)}$
3	$\frac{1}{\ln(3.5)}$
4	$\frac{1}{\ln(4.5)}$
5	$\frac{1}{\ln(5.5)}$.

$\Delta x = 1$

$$\int_1^5 \frac{1}{\ln(x+.5)}\,dx \approx \frac{1}{2}\left(\frac{1}{\ln(1.5)} + \frac{2}{\ln(2.5)} + \frac{2}{\ln(3.5)} + \frac{2}{\ln(4.5)} + \frac{1}{\ln(5.5)} \right)$$

≈ 4.0809

Problem 12

Let $f(x,y) = (\sin x + \cos y)^2$. Compute the partial derivatives of f with respect to x and y. Then:

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = \cdots$$

$\frac{\partial f}{\partial x} = 2(\sin x + \cos y)(\cos x)$

$\frac{\partial f}{\partial y} = 2(\sin x + \cos y)(-\sin y)$

A) $2(\sin x + \cos y)(\cos y + \sin x)$

B) $2(\sin x + \cos y)(\cos y - \sin x)$

C) $2(\sin x + \cos y)(\cos y + \cos x)$

D) $2(\sin x + \cos y)(\cos y - \cos x)$

E) $2(\sin x + \cos y)(\cos x + \sin y)$

F) $2(\sin x + \cos y)(\cos x - \sin y)$

G) $2(\sin x + \cos y)(-\cos x - \cos y)$

H) $2(\sin x + \cos y)(\cos x - \cos y)$

$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} =$

$2(\sin x + \cos y)(\cos x - \sin y)$

Problem 13

Calculate the iterated integral

$$\int_1^2 \int_0^2 \frac{y}{x}\, dy dx$$

A) $\ln 2$

B) $\frac{1}{2}$

C) $\frac{1}{2}\ln 2$

D) $2\ln 2$

E) 2

F) $\frac{2}{\ln 2}$

G) $\frac{1}{\ln 2}$

H) $\ln \frac{1}{2}$

$$\int_1^2 \int_0^2 \frac{y}{x}\, dy\, dx$$

$$\int_1^2 \frac{1}{x} \left[\frac{y^2}{2} \right]_0^2 dx$$

$$= \int_1^2 \frac{1}{x} (2)\, dx$$

$$= 2\ln(x) \Big|_1^2 = 2\ln(2)$$

Problem 14

Suppose you have an income stream that produces income of $K(t) = 1000e^{0.1t}$ at time t, and that as you receive the income you invest it at an interest rate of 5% for 10 years. What is the present value of this income stream?

A) \$12,874

B) \$12,924

C) \$12,974

D) \$13,024

E) \$13,074

F) \$13,124

G) \$13,174

H) \$13,224

$$P.V. = \int_0^{10} 1000 e^{.1t} e^{-.05t} dt.$$

$$= 1000 \int_0^{10} e^{.05t} dt$$

$$= (1000)(20) \left[e^{.05t} \right]_0^{10}$$

$$= 20{,}000 \left[e^{.5} - 1 \right],$$

$\approx 12{,}974$

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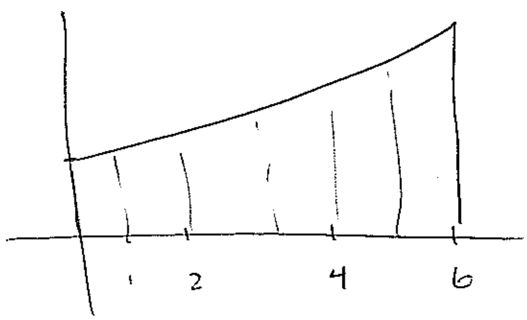
Written problem - Show your work

Problem 15

Use Simpson's Rule with $n = 3$ to approximate $\int_0^6 \sqrt{1+x^3}\,dx$ to 4 decimal places.



$f(x) = \sqrt{1+x^3}.$

$\Delta x = \dfrac{6-0}{3} = 2.$

$$\int_0^6 \sqrt{1+x^3}\,dx \approx \frac{\Delta x}{6}\left(f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6) \right)$$

$$= \frac{1}{3}\left(1 + 4\sqrt{2} + 2\sqrt{9} + 4\sqrt{28} + 2\sqrt{65} + 4\sqrt{126} + \sqrt{217} \right)$$

$$\approx 36.5261$$

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Written problem - Show your work

Problem 16

Compute

$$\int_0^\infty \frac{1}{(2x+3)^2}\,dx$$

or show that this integral diverges.

$$= \lim_{b\to\infty} \int_0^b \frac{dx}{(2x+3)^2}$$

$$= \lim_{b\to\infty} -\frac{1}{2}\left[\frac{1}{(2x+3)}\right]_0^b$$

$$= \lim_{b\to\infty} -\frac{1}{2}\left[\underbrace{\frac{1}{2b+3}}_{\to 0} - \frac{1}{3}\right]$$

$$= \frac{1}{6}.$$