Calculus II for the Life, Social and Managerial Sciences

Math 128 — Fall 2007

Final exam December 17

Name: **Student-ID:**

This exam contains 20 multiple choice questions, which count 5% each towards your total score. If your answer to a question is not exactly equal to the answers given, choose the closest answer. The last page of this exam displays a normal distribution table which you can use.

Problem 1

The highway department kept track of the number of accidents on a 10-mile stretch of a major highway every day during the third week of December. They found that there were:

1 day with no accidents
2 days with 1 accident
2 days with 2 accidents
No days with 3 accidents
1 day with 4 accidents
1 day with 5 accidents

Let X be the random variable that represents the number of accidents on a randomly chosen day in the third week of December.
Find E(X).

A) 1.94	**E)** 2.34
B) 2.04	**F)** 2.44
C) 2.14	**G)** 2.54
D) 2.24	**H)** 2.64

Problem 2

Evaluate

$$\int_0^{\sqrt{\pi}} x \sin(x^2 - \pi)\, dx$$

A) -2

B) -1

C) -0.5

D) 0

E) 0.5

F) 1

G) 2

H) π

$u = x^2 - \pi$

$du = 2x\,dx$

$= \frac{1}{2} \int_{-\pi}^{0} \sin(u)\, du.$

$= -\frac{1}{2} [\cos(u) \Big|_{-\pi}^{0}$

$= -\frac{1}{2} [1 - -1] = -1$

Problem 3

Suppose 4% of the students at a large university have red hair. You interview a number of students, one at a time.
Find the probability that the fifth student interviewed is the first to have red hair.

$\left(\frac{96}{100}\right)^4 \left(\frac{4}{100}\right) = 0.034$

A) 0.024

B) 0.034

C) 0.044

D) 0.054

E) 0.120

F) 0.170

G) 0.220

H) 0.270

Problem 4

In the situation of Problem 3, find the probability that exactly two students with red hair are among the first five students interviewed.

A) 0.001

B) 0.002

C) 0.004

D) 0.007

E) 0.008

F) 0.009

G) 0.012

(H) 0.014

$$\binom{5}{2}\left(\frac{96}{100}\right)^3\left(\frac{4}{100}\right)^2$$

Total Number of orderings — 3 non-Red Hair — 2 Red Hair

$$\approx \frac{5\cdot 4}{2\cdot 1}(.0014)$$

$$= .014$$

Problem 5

$$\int (x-3)e^{-x}\,dx = \cdots$$

$u = x-3 \quad dv = e^{-x}\,dx$

$du = dx \quad v = -e^{-x}$

$-(x-3)e^{-x} + \int e^{-x}\,dx.$

$= -(x-3)e^{-x} - e^{-x} + C$

$= -e^{-x}(x-3+1) + C$

$= -e^{-x}(x-2) + C.$

A) $(x-1)e^{-x} + C$

B) $(x-2)e^{-x} + C$

C) $(x-3)e^{-x} + C$

D) $(x-4)e^{-x} + C$

E) $-(x-1)e^{-x} + C$

F) $-(x-2)e^{-x} + C$

G) $-(x-3)e^{-x} + C$

H) $-(x-4)e^{-x} + C$

Problem 6

Suppose X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2}e^{-x} + xe^{-x^2} & \text{if } x \geq 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Compute the cumulative distribution function $F(x)$ (for $x \geq 0$).

A) $\frac{1}{2}e^{-x} - e^{-x^2}$

B) $\frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2}$

C) $-\frac{1}{2}e^{-x} - e^{-x^2}$

D) $-\frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2}$

E) $1 + \frac{1}{2}e^{-x} - e^{-x^2}$

F) $1 + \frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2}$

G) $1 - \frac{1}{2}e^{-x} - e^{-x^2}$

H) $1 - \frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2}$

If $x \geq 0$,

$$F(x) = \int_0^x \frac{1}{2}e^{-t} + te^{-t^2}\,dt$$

$$= -\frac{1}{2}e^{-t} - \frac{1}{2}e^{-t^2}\Big|_0^x$$

$$= -\frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2} - \Big(\underbrace{-\frac{1}{2} - \frac{1}{2}}_{-1}\Big).$$

$$= 1 - \frac{1}{2}e^{-x} - \frac{1}{2}e^{-x^2}$$

Problem 7

Suppose you have an income stream that produces income of $K(t) = 50e^{-0.08t}$ thousand dollars per year at time t, and that as you receive the income you invest it at an interest rate of 12% for 4 years. What is the present value of this income stream?

A) \$135,784

B) \$136,098

C) \$136,412

D) \$136,726

E) \$137,040

F) \$137,354

G) \$137,668

H) \$137,982

$$\int_0^4 50e^{-.08t} \, e^{-.12t} \, dt.$$

$$= 50 \int_0^4 e^{-.2t} \, dt.$$

$$= 50 \int_0^4 e^{-\frac{1}{5}t} \, dt$$

$$= 50 \, (-5) \left[e^{-\frac{1}{5}t} \right|_0^4$$

$$= -250 \left[e^{-4/5} - 1 \right].$$

$\sim$ 137,668 Thousand.

Problem 8

Suppose X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{4}{x^5} & \text{if } x \geq 1 \\ 0 & \text{if } x \leq 1 \end{cases}$$

Compute $\Pr(2 \leq X \leq 3)$.

$$\int_2^3 \frac{4}{x^5} dx = -\frac{1}{x^4} \Big|_2^3 = -\frac{1}{3^4} + \frac{1}{2^4} \approx .0502$$

A) 0.0482

B) 0.0492

C) 0.0502

D) 0.0512

E) 0.0522

F) 0.0532

G) 0.0542

H) 0.0552

Problem 9

Evaluate the improper integral

$$\int_1^\infty \frac{2x}{(x^2+1)^4}\,dx$$

A) $\frac{1}{32}$
B) $\frac{1}{24}$
C) $\frac{1}{12}$
D) $\frac{1}{8}$
E) $\frac{1}{6}$
F) $\frac{1}{4}$
G) $\frac{1}{3}$
H) 1

$$= \lim_{B\to\infty} \int_1^B \frac{2x}{(x^2+1)^4}\,dx$$

$$\left(\begin{array}{l} u = x^2+1 \\ du = 2x\,dx. \end{array} \right)$$

$$= \lim_{B\to\infty} -\frac{1}{3}\left[\frac{1}{(x^2+1)^3}\right]_1^B$$

$$= \lim_{B\to\infty} -\frac{1}{3}\left[\frac{1}{(B^2+1)^3} - \frac{1}{8}\right].$$

$$= \frac{1}{24}$$

Problem 10

Suppose X is a continuous random variable with probability density function

$$f(x) = \begin{cases} 6(x - x^2) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find E(X) and Var(X).

A) E(X) = 0.5, Var(X) = 0.05

B) E(X) = 0.5, Var(X) = 0.3

C) E(X) = 0.5, Var(X) = 0.6

D) E(X) = 0.6, Var(X) = 0.05

E) E(X) = 0.6, Var(X) = 0.3

F) E(X) = 0.6, Var(X) = 0.6

G) E(X) = 0.75, Var(X) = 0.3

H) E(X) = 0.75, Var(X) = 0.6

$$E(X) = \int_0^1 x f(x)dx = \int_0^1 6(x^2 - x^3)\,dx = 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{6}{12} = \frac{1}{2}.$$

$$Var(X) = \int_0^1 x^2 f(x)dx - \mu^2 = \int_0^1 6(x^3 - x^4)\,dx - \frac{1}{4}$$

$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right|_0^1 - \frac{1}{4} = 6\left[\frac{1}{4} - \frac{1}{5}\right] - \frac{1}{4}$$

$$= 6\left[\frac{1}{20}\right] - \frac{1}{4}$$

$$= .05$$

Problem 11

Solve the differential equation

$$y' = \frac{t^3}{y^2}$$

$$\int y^2\,dy = \int t^3\,dt$$

A) $y = \sqrt[3]{\frac{3}{4}t^4 + C}$

B) $y = \sqrt[3]{\frac{3}{4}t^4} + C$

C) $y = \sqrt[3]{\frac{1}{4}t^4 + C}$

D) $y = \sqrt[3]{\frac{1}{4}t^4} + C$

E) $y = \sqrt{\frac{3}{4}t^4 + C}$

F) $y = \sqrt{\frac{3}{4}t^4} + C$

G) $y = \sqrt{\frac{1}{4}t^4 + C}$

H) $y = \sqrt{\frac{1}{4}t^4} + C$

$$\frac{y^3}{3} = \frac{t^4}{4} + C.$$

$$y^3 = \frac{3}{4}t^4 + C'$$

$$y = \sqrt[3]{\frac{3}{4}t^4 + C}$$

Problem 12

The length of time (in minutes) that it takes a laboratory rat to traverse a maze is an exponential random variable X. Suppose that a rat needs an average time of 3 minutes to complete the maze.
What is the probability that a random rat is not able to traverse the maze within 4 minutes?

pdf:

$E(X) = \frac{1}{k} = 3. \Rightarrow k = \frac{1}{3}.$

$f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

A) 0.2267

B) 0.2390

C) 0.2513

D) 0.2636

E) 0.2759

F) 0.2882

G) 0.3005

H) 0.3128

$P(X \geq 4) = \frac{1}{3} \int_4^{\infty} e^{-\frac{1}{3}x} dx$

$= \lim_{T \to \infty} - \left[e^{-\frac{1}{3}x} \right|_4^T$

$= - \lim_{T \to \infty} \left[\frac{1}{e^{\frac{1}{3}T}} - \frac{1}{e^{4/3}} \right]$ (first term $\to 0$.)

$\approx \frac{1}{e^{4/3}} \approx .2636.$

Problem 13

Solve the initial value problem

$$ty' + y = \sqrt{t}, \quad y(9) = 1$$

$$y' + \frac{1}{t} y = \frac{1}{\sqrt{t}}.$$

A) $y = 2\sqrt{t} - \frac{9}{t}$

B) $y = 2\sqrt{t} + \frac{9}{t}$

C) $y = 2\sqrt{t} - \frac{27}{t}$

D) $y = 2\sqrt{t} + \frac{27}{t}$

E) $y = \frac{2}{3}\sqrt{t} - \frac{9}{t}$

F) $y = \frac{2}{3}\sqrt{t} + \frac{9}{t}$

G) $y = \frac{2}{3}\sqrt{t} - \frac{27}{t}$

H) $y = \frac{2}{3}\sqrt{t} + \frac{27}{t}$

$$A(x) = \int \frac{1}{t}\, dt = \ln(t)$$

$$e^{A(x)} = t$$

$$ty' + y = \sqrt{t}.$$

$$(yt)' = \sqrt{t}.$$

$$yt = \int \sqrt{t}\, dt = \frac{2}{3} t^{3/2} + c$$

$$y = \frac{2}{3} t^{1/2} + \frac{c}{t}.$$

$$1 = y(9) = \frac{2}{3}(3) + \frac{c}{9}$$

$$= +2 + \frac{c}{9}.$$

$$(-1)9 = c.$$

$$c = -9.$$

$$y = \frac{2}{3} t^{1/2} + \frac{-9}{t}.$$

Problem 14

The amount of garbage a household in the United States discards in a month follows a normal distribution with an average of 225 pounds and a standard deviation of 19 pounds.
Find the percentage of households that discard between 250 and 275 pounds of garbage per month.

A) 8.57 %

B) 8.74 %

C) 8.91 %

D) 9.08 %

E) 9.25 %

F) 9.42 %

G) 9.59 %

H) 9.76 %

$P(250 \le X \le 275)$.

$= P\left(\frac{250-225}{19} \le Z \le \frac{275-225}{19}\right)$

$= P(1.3157 \le Z \le 2.6315)$.

$= P(Z \le 2.6315) - P(Z \le 1.3157)$

$= .9957 - .9066$

$= .0891$

$= 8.91\%$.

Problem 15

Use Euler's method with $n = 3$ on the interval $0 \le t \le 1.5$ to approximate the solution $y(t)$ to the initial value problem

$$y' = 2t - y + 1, \quad y(0) = 5$$

Give as your answer the approximation of $y(1.5)$.

t	$y(t)$	$y'(t)$	Approximating line
0	5	$-5+1=-4$	$y = 5 + -4t$.
.5	+3	$1-3=-2$	$y-3 = -2(t-\frac{1}{2})$
1.0	$3+-1=2$	2-	
1.5			

A) 1.25

B) 1.5

C) 1.75

D) 2

E) 2.25

F) 2.5

G) 2.75

H) 3

t	$y(t)$	$y'(t)$	Approximating line
0	5	$-5+1=-4$	$y = 5 - 4t$
.5	3	$1-3+1=-1$	$y - 3 = -1(t-\frac{1}{2})$
1.0	$3-\frac{1}{2}=\frac{5}{2}$	$2-\frac{5}{2}+1=\frac{1}{2}$.	$y = \frac{5}{2} + \frac{1}{2}(t-1)$.
1.5	2.75		

Problem 16

Suppose that the number of shark attacks in coastal waters around the Unites States follows a Poisson distribution with a mean of 0.24 shark attacks per day.
Find the probability that on a randomly selected day, there will be exactly one shark attack.

A) 0.137

B) 0.150

C) 0.163

D) 0.176

E) 0.189

F) 0.202

G) 0.215

H) 0.228

$E(X) = \lambda = .24$

So, $P(X = n) = \frac{\lambda^n e^{-\lambda}}{n!}$

$P(X = 1) = \frac{(.24)\, e^{-.24}}{1!}$

$\approx .1887$

Problem 17

In the situation of Problem 16, what is the probability that during a randomly selected week, there are no shark attacks in U.S. coastal waters?

$P(X=0)$ on day 1.

AND $P(X=0)$ on day 2

AND ...

$P(X=0)$ on day 7.

A) 0.1679

B) 0.1716

C) 0.1753

D) 0.1790

E) 0.1827

F) 0.1864

G) 0.1901

H) 0.1938

But $P(X=0) = e^{-.24}$

so, $\left(e^{-.24}\right)^7 = .18637\ldots$

Problem 18

Find the second degree Taylor polynomial of $f(x) = \ln(\sin x)$ at $x = \frac{\pi}{2}$.

A) $1 + (x - \frac{\pi}{2}) - \frac{1}{2}(x - \frac{\pi}{2})^2$

B) $1 - \frac{1}{2}(x - \frac{\pi}{2})^2$

C) $(x - \frac{\pi}{2}) - \frac{1}{2}(x - \frac{\pi}{2})^2$

D) $-\frac{1}{2}(x - \frac{\pi}{2})^2$

E) $1 + (x - \frac{\pi}{2}) + \frac{1}{2}(x - \frac{\pi}{2})^2$

F) $1 + \frac{1}{2}(x - \frac{\pi}{2})^2$

G) $(x - \frac{\pi}{2}) + \frac{1}{2}(x - \frac{\pi}{2})^2$

H) $\frac{1}{2}(x - \frac{\pi}{2})^2$

$f(\frac{\pi}{2}) = \ln(1) = 0$

$f'(x) = \frac{1}{\sin x} \cos x = \cot x,$

$f'(\pi/2) = 0,$

$f''(x) = -\csc^2(x) =$

$f''(\pi/2) = -1$

$P_2(x) = 0 + 0 + -\frac{1}{2!}(x - \pi/2)^2.$

Problem 19

Find the sum of the geometric series $9.5 - 8.55 + 7.695 - 6.9255 + \cdots$

$$= 9.5\left(1 + (-.9) + (-.9)^2 + \cdots\right)$$

$$= 9.5\left(\frac{1}{1+.9}\right)$$

$$= 5$$

A) 2.5

B) 3.75

C) 5

D) 6.25

E) 7.5

F) 8.75

G) 10

H) This series diverges.

Problem 20

Use Newton's method to approximate a zero of $f(x) = x^3 + 3x + 1$. Let $x_0 = 0$ be the initial approximation. Give as your answer the second approximation x_2.

A) -0.322

B) -0.325

C) -0.330

D) -0.333

E) -0.340

F) -0.345

G) -0.350

H) -0.355

Iteration	$f'(x)$	Linear Approx. for f	Zero of Approximation
0	$3x^2$		

$f'(x) = 3x^2 + 3$.

$$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$$

$$= 0 - \frac{1}{3}$$

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

$$= -\frac{1}{3} - \frac{\left(-\frac{1}{27} - 1 + 1\right)}{\frac{1}{3} + 3}$$

$= -.322\ldots$